AN ALTERNATIVE TO NONLINEAR ESTIMATION IN GUN FIRE CONTROL SYSTEMS

Robert Keith Brands

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by

Robert Keith Brands

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against a set of test trajectories that include constant-velocity and maneuvering target profiles. The results of the Monte Carlo simulation of these filters are compared with the results obtained from a filter derived from a Cartesian Coordinate state vector. Switch-on-range adaptive filters were developed from the two basic models and evaluated by Monte Carlo methods. The results of the simulations of the adaptive filters with the two different state vectors are compared.

An Alternative to Nonlinear Estimation
in
Gun Fire Control Systems

by

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Lieutenant, United States Navy
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TABLE OF SYMBOLS AND ABBREVIATIONS

$\underset{\sim}{x}(K)$	n-dimensional state vector at time $t = KT$.
$\underset{\sim}{w}(K)$	p-dimensional random forcing vector at time $t = KT$.
$\underset{\sim}{\Phi}(K+1,K)$	(n x n)-dimensional state transition matrix.
$\underset{\sim}{\Gamma}(K)$	(n x p)-dimensional random forcing distribution matrix.
$\underset{\sim}{z}(K)$	m-dimensional vector of observations at time $t = KT$.
$\underset{\sim}{H}(K)$	(m x n)-dimensional observation matrix.
$\underset{\sim}{v}(K)$	m-dimensional measurement noise vector at time $t = KT$.
T	sampling period.
K	a non-negative integer denoting the current sample.
E	expectation operator.
δ_{kj}	Kronecker Delta Function.
$\underset{\sim}{Q}(K)$	(p x p)-dimensional covariance of random forcing matrix at time $t = KT$.
$\underset{\sim}{R}(K)$	(m x m)-dimensional covariance of measurement noise at time $t = KT$.
$\hat{\underset{\sim}{x}}(K/K)$	n-dimensional vector of state estimates at time $t = KT$, given measurements up to and including time $t = KT$.
$\hat{\underset{\sim}{x}}(K/K-1)$	n-dimensional vector of the one-step state prediction.
$\underset{\sim}{G}(K)$	(n x m)-dimensional Kalman Gain matrix at time $t = KT$.
$\underset{\sim}{P}(K/K)$	(n x n)-dimensional covariance of estimation error matrix.
$\underset{\sim}{P}(K+1/K)$	(n x n)-dimensional one-step prediction error covariance matrix.

$\underset{\sim}{I}$	(n x n)-dimensional identity matrix.
$[\ ]^T$	matrix(vector) transpose.
$[\ ]^{-1}$	matrix inverse.

I. INTRODUCTION

A. AN ALTERNATIVE TO NONLINEAR ESTIMATION IN GUN FIRE CONTROL SYSTEMS

The increased performance characteristics of modern aircraft and missiles has made apparent the need for more effective fire control systems. With the introduction of digital fire control computers, Kalman filtering techniques offer an attractive means of improving the effectiveness of a fire control system. Indeed, several recent studies conducted at the Naval Postgraduate School [Refs. 1-6] have specifically addressed the application of Kalman filtering techniques to fire control systems. It is interesting to note that the references cited above all have one distinctive feature in common; they use Cartesian Coordinate state vectors. A Cartesian Coordinate state vector offers the attraction of linear plant dynamics for a constant-velocity target but has the disadvantage of nonlinear measurement equations when used in a fire control system that measures the target's position in spherical coordinates. One possible solution to the nonlinear measurement problem is to process the sensor measurements through a prefilter as in Figure 1. The output of the sensor consists of noise corrupted measurements of the target's position in spherical coordinates

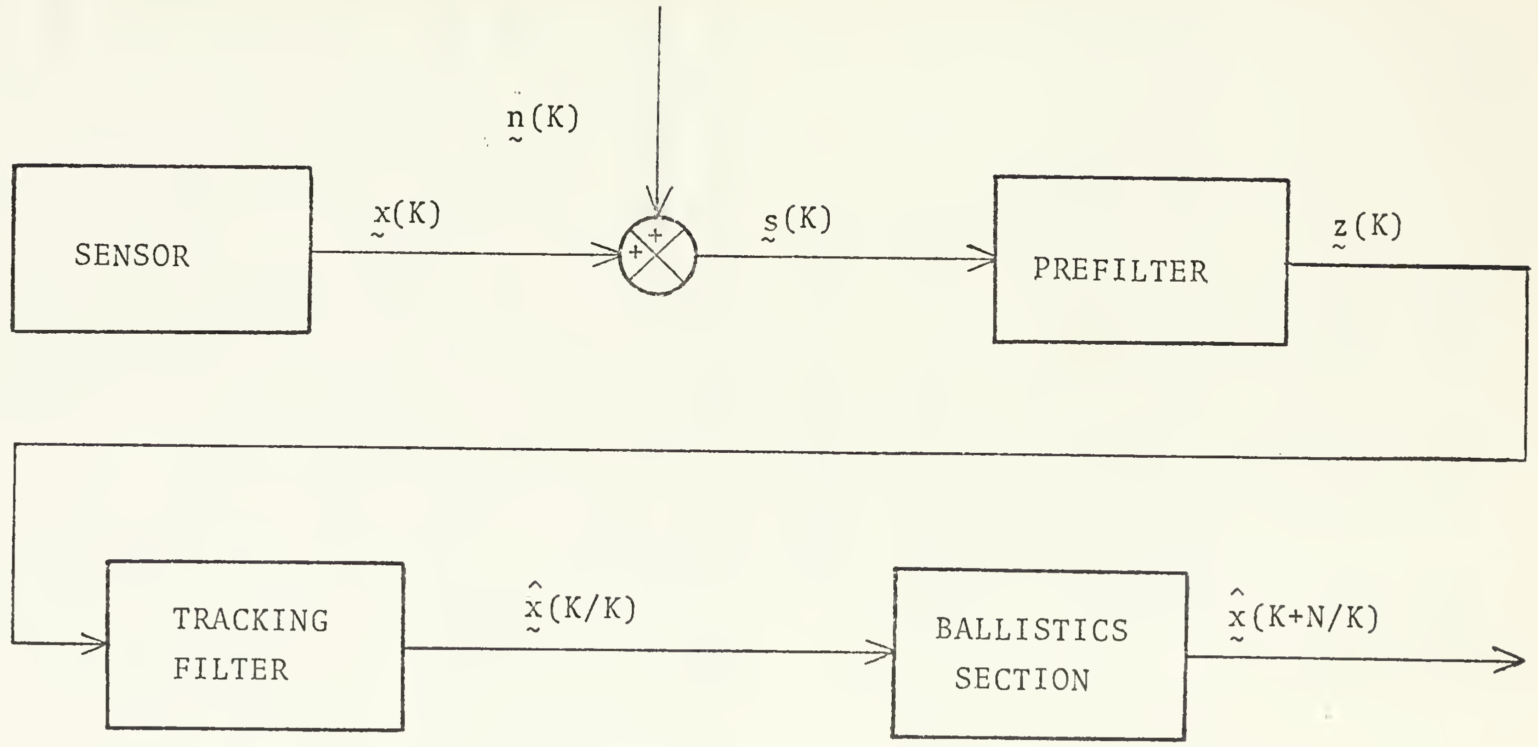

Fig. 1. FIRE CONTROL SYSTEM SIMPLIFIED BLOCK DIAGRAM

$$
\begin{aligned}
s_1(K) &= r(K) + n_1(K) \\
s_2(K) &= \phi(K) + n_2(K) \qquad (1.1) \\
s_3(K) &= \theta(K) + n_3(K)
\end{aligned}
$$

which serve as inputs to the prefilter. The appropriate coordinate transformation equations from Figure 2 are

$$
\begin{aligned}
x(K) &= r(K)\sin\phi(K)\cos\theta(K) \\
y(K) &= r(K)\cos\phi(K)\cos\theta(K) \qquad (1.2) \\
z(K) &= r(K)\sin\theta(K)
\end{aligned}
$$

where it has been shown [Ref. 3] that $\underset{\sim}{v}(K)$, the measurement noise process presented to the tracking filter is a non-linear, position-dependent function of the variances of the range, bearing, and elevation errors. Clearly then, the measurements become correlated in the coordinate conversion process. Full implementation of such a filtering scheme could conceivably involve much on-line computation since the statistics for $\underset{\sim}{v}(K)$ are no longer stationary. Another distinct disadvantage of the prefiltering method lies in the errors generated due to the added finite precision arithmetic of the coordinate conversion process. Such errors may be qualitatively described as a noise process whose statistical properties are difficult at best to identify. An excellent treatment of this specific topic is found in Gelb [Ref. 7].

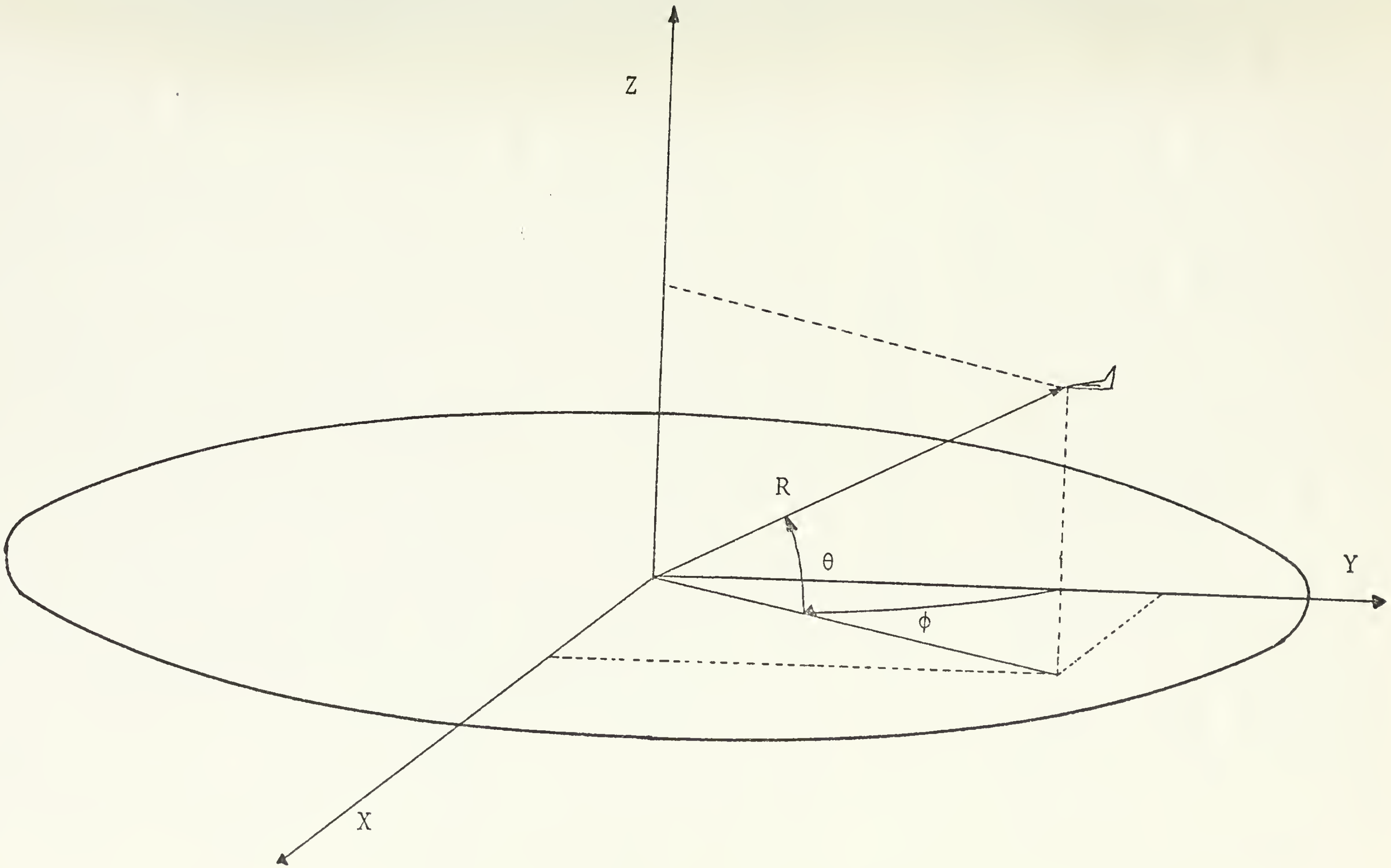

Fig. 2. THE SPHERICAL COORDINATE SYSTEM

Another approach to the nonlinear measurement process is to linearize the measurement equations about some reference or nominal trajectory. Unfortunately, such an approach is usually accomplished by the implementation of an extended or iterative extended Kalman filter with the attendant increase in on-line computing requirements.

Consider the filtering problem with a spherical coordinate state vector

$$x(K) = [r,\dot{r},\ddot{r},\phi,\dot{\phi},\ddot{\phi},\theta,\dot{\theta},\ddot{\theta}]^T \qquad (1.4)$$

Such a state vector has the advantage of linear measurement equations eliminating the correlation and position dependence of the measurement noise process. The disadvantage of a spherical state vector lies in the coupled and nonlinear system-target-dynamics, even for a constant-velocity target. Once again, the implementation of this filter would normally be with an extended or iterative extended Kalman filter. This thesis proposes a filter having a spherical coordinate state vector with assumed linear system dynamics. In view of the fact that any system model is an approximation to a physical system, this seems to be a reasonable assumption. Implementation of this would be a great deal simpler than the previously mentioned filters. A filter designed on this basis would then of necessity be sub-optimal. However, given acceptable performance, the sub-optimal filter might be preferred to the optimal nonlinear filter. Faced then,

with a sub-optimal filter due to modeling inaccuracies, a technique must be found to compensate for the unmodeled nonlinear properties of the system. Jazwinski [Ref. 8] has proposed two methods for the compensation of inaccurate a priori knowledge of the system. The first involves a fictitious noise input to the system, while the second overweights the most recent data. The net result of either method is an increase in the filter bandwidth, making the filter more responsive to system transients but increasing its sensitivity to the measurement noise process. The fictitious noise input method was chosen as the most promising and was used in this thesis. So that some quantifiable measure of the feasibility of these filters is possible, this thesis paralleled Parr's work [Ref. 4] and comparisons are made with his results.

B. LINEAR FILTERING THEORY

The system studied in this thesis is composed of a target and a sensor which gives noise-corrupted measurements of the target's position in spherical coordinates. The system may be approximated by the linear difference equations

$$\begin{aligned} \underset{\sim}{x}(K+1) &= \underset{\sim}{\phi}(K+1,K)\underset{\sim}{x}(K) + \underset{\sim}{\Gamma}(K)\underset{\sim}{w}(K) \\ \underset{\sim}{z}(K) &= \underset{\sim}{H}(K)\underset{\sim}{x}(K) + \underset{\sim}{v}(K) \end{aligned} \qquad (1.5)$$

The statistics assumed for the stochastic processes $\underset{\sim}{w}(K)$

and $\underset{\sim}{v}(K)$ are

$$E[\underset{\sim}{w}(K)] = E[\underset{\sim}{v}(K)] = \underset{\sim}{0} \quad \text{for all } K \tag{1.6}$$

$$E[\underset{\sim}{v}(k)\underset{\sim}{v}^T(j) = \underset{\sim}{R}(K)\delta_{kj} \tag{1.7}$$

$$E[\underset{\sim}{w}(k)\underset{\sim}{w}^T(j) = \underset{\sim}{Q}(K)\delta_{kj} \tag{1.8}$$

$$E[\underset{\sim}{w}(K)\underset{\sim}{v}^T(K)] = E[\underset{\sim}{v}(K)\underset{\sim}{w}^T(K)] = \underset{\sim}{0} \quad \text{for all } K \tag{1.9}$$

$$E[\underset{\sim}{w}(K)\underset{\sim}{x}^T(0)] = E[\underset{\sim}{x}(0)\underset{\sim}{w}^T(K)] = \underset{\sim}{0} \quad \text{for all } K \tag{1.10}$$

$$E[\underset{\sim}{v}(K)\underset{\sim}{x}^T(0)] = E[\underset{\sim}{x}(0)\underset{\sim}{v}^T(K)] = \underset{\sim}{0} \quad \text{for all } K \tag{1.11}$$

The linear, discrete Kalman filter is characterized by the difference equations

$$\hat{\underset{\sim}{x}}(K/K) = \hat{\underset{\sim}{x}}(K/K-1) + \underset{\sim}{G}(K)[\underset{\sim}{z}(K) - \underset{\sim}{H}(K)\hat{\underset{\sim}{x}}(K/K-1)] \tag{1.12}$$

with the one-step prediction given by

$$\hat{\underset{\sim}{x}}(K/K-1) = \underset{\sim}{\phi}(K,K-1)\hat{\underset{\sim}{x}}(K-1/K-1) \tag{1.13}$$

The Kalman Gain matrix, $\underset{\sim}{G}(K)$, is found by solving the following equations

$$\underset{\sim}{G}(K) = \underset{\sim}{P}(K/K-1)\underset{\sim}{H}^T(K)[\underset{\sim}{H}(K)\underset{\sim}{P}(K/K-1)\underset{\sim}{H}^T(K) + \underset{\sim}{R}(K)]^{-1} \tag{1.14}$$

$$\underset{\sim}{P}(K/K) = [\underset{\sim}{I} - \underset{\sim}{G}(K)\underset{\sim}{H}(K)]\underset{\sim}{P}(K/K-1) \tag{1.15}$$

$$\begin{aligned}\underset{\sim}{P}(K+1/K) = {} & \underset{\sim}{\phi}(K+1,K)\underset{\sim}{P}(K/K)\underset{\sim}{\phi}^T(K+1,K) + \\ & \underset{\sim}{\Gamma}(K)\underset{\sim}{Q}(K)\underset{\sim}{\Gamma}^T(K)\end{aligned} \tag{1.16}$$

To start the gain calculations, the matrix $\underset{\sim}{P}(K/K-1)$ must

be initialized. Taking the standard approach, $\underset{\sim}{P}(0/-1)$ is given by

$$\underset{\sim}{P}(0/-1) = E[(\underset{\sim}{x}(0) - \underset{\sim}{\overline{x}}_0)(x(0) - \underset{\sim}{\overline{x}}_0)^T] \qquad (1.17)$$

where

$$E[\underset{\sim}{x}(0)] = \underset{\sim}{\overline{x}}_0 \qquad (1.18)$$

An unbiased estimator is obtained if the filter is started by setting

$$\underset{\sim}{\hat{x}}(0/-1) = \underset{\sim}{\overline{x}}_0 \qquad (1.19)$$

II. DEVELOPMENT OF THE FILTER MODELS

A. CONSTANT-VELOCITY ($1/S^2$) MODEL

The first filter tested was based on a dynamic system model that assumes linear constant-velocity target motion in each of three coordinate directions.

The state transition matrix $\underset{\sim}{\phi}(K+1,K)$ and the random forcing input distribution matrix $\underset{\sim}{\Gamma}(K)$ are

$$\underset{\sim}{\phi}(K+1,K) = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.1}$$

and

$$\underset{\sim}{\Gamma}(K) = \begin{bmatrix} \frac{T^2}{2} & 0 & 0 \\ T & 0 & 0 \\ 0 & \frac{T^2}{2} & 0 \\ 0 & T & 0 \\ 0 & 0 & \frac{T^2}{2} \\ 0 & 0 & T \end{bmatrix} \tag{2.2}$$

where T is the sampling period.

Since the state vector is expressed in spherical coordinates, the measurement process is linear and is given by

$$\underset{\sim}{z}(K) = \underset{\sim}{H}(K)\underset{\sim}{x}(K) + \underset{\sim}{v}(K) \qquad (2.3)$$

The measurement noise processes for the individual coordinate directions are assumed to be independent processes. The covariance of measurement noise matrix $\underset{\sim}{R}(K)$ is diagonal and is given by

$$\underset{\sim}{R}(K) = \begin{bmatrix} \sigma_R^2 & & \underset{\sim}{0} \\ & \sigma_B^2 & \\ \underset{\sim}{0} & & \sigma_E^2 \end{bmatrix} \qquad (2.4)$$

Because of its simplicity the constant-velocity model cannot realistically be expected to perform well against all types of target trajectories and may not perform acceptably against maneuvering targets. The performance of the constant-velocity model filter can be improved by the use of the system random input covariance matrix $\underset{\sim}{Q}(K)$. Indeed, one of the functions of the $\underset{\sim}{Q}(K)$ matrix is to compensate for the inadequacies of a simplified model, and for this specific thesis, $\underset{\sim}{Q}(K)$ can also partially account for the system nonlinearities not explicitly included in the system model. The subject of the sensitivity of the

filter's N-step prediction performance to the $\underset{\sim}{Q}(K)$ matrix forms the basis for the simulations of Chapter III.

Initialization of the filter requires a priori knowledge of the statistics of the system initial conditions so that the gain calculations may be started. For any fixed coordinate system, it is reasonable to consider the mean of all possible system initial conditions as zero or

$$E[\underset{\sim}{x}(0)] = \bar{\underset{\sim}{x}}_0 = \underset{\sim}{0} \tag{2.5}$$

It has been shown [Refs. 7 and 9] that under proper initialization, the filter will remain unbiased for $K \geq 0$, i.e.,

$$E[\underset{\sim}{e}(K/K)] = \underset{\sim}{0} \quad \text{for all } K \tag{2.6}$$

where

$$\underset{\sim}{e}(K/K) = \hat{\underset{\sim}{x}}(K/K) - \underset{\sim}{x}(K) \tag{2.7}$$

The unbiased condition is satisfied if

$$\hat{\underset{\sim}{x}}(0/-1) = \bar{\underset{\sim}{x}}_0 \tag{2.8}$$

Initialization of the covariance of estimation error matrix requires a somewhat different approach. Following Parr [Ref. 4] it is quantitatively shown that if the diagonal elements of the initial covariance of estimation error matrix are made very large, the net effect is that the first position estimate is made identically equal to

the first system measurement. In this manner the system is forced into tracking the target rapidly. For all simulations conducted then, it was assumed that

$$\underset{\sim}{P}(0/-1) = \begin{bmatrix} 10^8 & & \underset{\sim}{0} \\ & \ddots & \\ \underset{\sim}{0} & & 10^8 \end{bmatrix} \tag{2.9}$$

B. CORRELATED RANDOM ACCELERATION MODEL

The constant-velocity model discussed in the previous section views any target maneuvers as small perturbations about a nominal, constant-velocity trajectory. This tacitly assumes that any target maneuvers are uncorrelated in time, or from one measurement to the next. Target maneuvers will generally take place over a period of n measurements and the magnitude of n is dependent on the level of the target's maneuver, i.e., for a slow turn n is relatively large whereas for an evasive type maneuver n is smaller. The fact that the accelerations occur over a finite period of time correlates the target's accelerations (maneuvers) in time. One approach to modeling the effects of time-correlated target maneuvers is to use a constant-velocity plant driven by correlated noise obtained from a low-pass filter of the form

$$T(s) = \frac{\alpha}{(s + \alpha)} \tag{2.10}$$

driven by white noise. The constant α is interpreted as the time constant or degree of time correlation of the target's maneuver.

For one dimension, the overall model transfer function is

$$T(s) = \frac{\alpha}{s^2(s + \alpha)} \tag{2.11}$$

as depicted in Figure 3.

For the three-dimensional case, the filter is ninth order and has the state vector

$$\underset{\sim}{x}(K) = [r, \dot{r}, \ddot{r}, \phi, \dot{\phi}, \ddot{\phi}, \theta, \dot{\theta}, \ddot{\theta}]^T \tag{2.12}$$

The correlated random acceleration model is discussed in detail by McKinley [Ref. 5] and Singer [Ref. 10]. From the cited references, the (3 x 3) state transition matrix $\underset{\sim}{\phi}_i(T)$ and the (3 x 1) random forcing input vector $\underset{\sim}{\Gamma}_i(T)$ are for the single dimension with assumed linear system dynamics

$$\phi_i(T) = \begin{bmatrix} 1.0 & T & \frac{T}{\alpha} - \frac{1}{\alpha^2}(1 - e^{-\alpha T}) \\ 0.0 & 1.0 & \frac{1}{\alpha}(1 - e^{-\alpha T}) \\ 0.0 & 0.0 & e^{-\alpha T} \end{bmatrix} \tag{2.13}$$

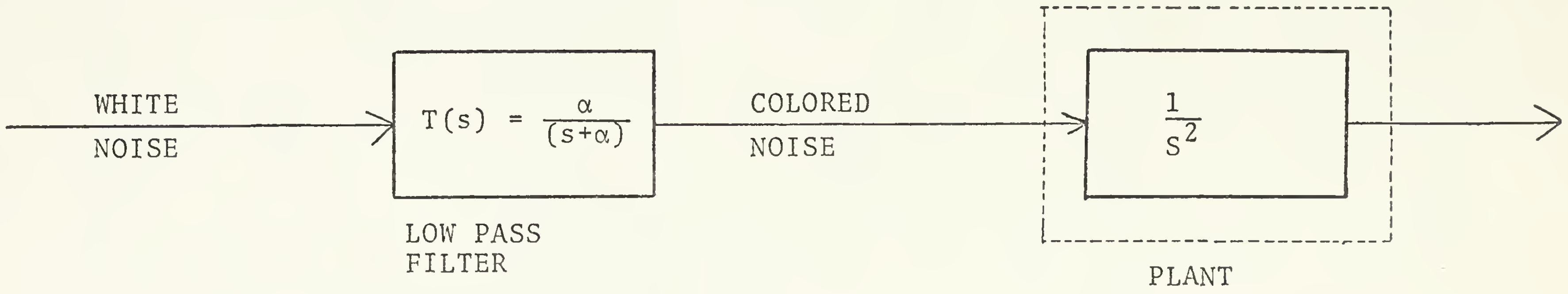

Fig. 3. CONSTANT-VELOCITY PLANT DRIVEN BY CORRELATED NOISE

and

$$\underset{\sim}{\Gamma}_i(T) = \begin{bmatrix} \frac{T^2}{2} - \frac{T}{\alpha} + \frac{1}{\alpha^2}(1 - e^{-\alpha T}) \\ T - \frac{1}{\alpha}(1 - e^{-\alpha T}) \\ (1 - e^{-\alpha T}) \end{bmatrix} \qquad (2.14)$$

For the ninth-order filter the state transition matrix $\underset{\sim}{\phi}(K+1,K)$ is formed by

$$\underset{\sim}{\phi}(K+1,K) = \begin{bmatrix} \underset{\sim}{\phi}_r(T) & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{\phi}_B(T) & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{\phi}_E(T) \end{bmatrix} \qquad (2.15)$$

where $\underset{\sim}{\phi}_r(T)$, $\underset{\sim}{\phi}_B(T)$, and $\underset{\sim}{\phi}_E(T)$ are individually given by $\underset{\sim}{\phi}_i(T)$ in Equation (2.13). In a similar manner, for the ninth-order filter the three-dimensional system matrix $\underset{\sim}{\Gamma}(K)$ is given by

$$\underset{\sim}{\Gamma}(K) = \begin{bmatrix} \underset{\sim}{\Gamma}_r(T) & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{\Gamma}_B(T) & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{\Gamma}_E(T) \end{bmatrix} \qquad (2.16)$$

where $\underset{\sim}{\Gamma}_r(T)$, $\underset{\sim}{\Gamma}_B(T)$, $\underset{\sim}{\Gamma}_E(T)$ are individually given by $\underset{\sim}{\Gamma}_i(T)$ in Equation (2.14).

The covariance of measurement noise matrix $\underset{\sim}{R}(K)$ is the same as given in the preceding section by Equation (2.4). Initialization of the filter is carried out in exactly the same fashion as described in the previous section by Equations (2.5), (2.8), and (2.9). As in the preceding section, $\underset{\sim}{Q}(K)$, the covariance of the random forcing input white noise process presented to the coloring filter, was used to compensate for the unmodeled dynamic nonlinearities. The sensitivity of the filter's N-step prediction performance to the $\underset{\sim}{Q}(K)$ matrix selected is treated in Chapter III.

In the interest of saving computer time, the correlated random acceleration model was tested using a single value for α, the maneuver time constant. Since Parr [Ref. 4] reported that the best overall performance in his simulations was obtained from a value of $\alpha = 0.5$, that value was used in the study of this model. Additionally, the maneuver time constant α, was fixed at the same value for each of the three coordinate directions, though this constraint need not necessarily be imposed on the system.

III. RESULTS OF THE MONTE CARLO SIMULATION OF THE PROPOSED LINEAR FILTERS

A. EXPERIMENTAL PROCEDURE

Monte Carlo simulation techniques were used in the optimization and testing of the filters developed in Chapter II. A listing and description of the FORTRAN program used is given in Appendix A. All results are the products of 100-member ensemble simulations. Test trajectories 1-8 were used in the optimization and testing process and Appendix B gives a description of the profile for each trajectory. The profiles of trajectories 1, 2, 3, and 4 are identical to the short-range constant-velocity, short-range maneuvering, long-range constant-velocity, and long-range maneuvering profiles, respectively, used by Parr [Ref. 4]. Trajectories 5-8 were included to examine the filter's performance against more severe target maneuvers.

The elements of $\underset{\sim}{R}(K)$, the covariance of measurement noise process matrix, were fixed for all simulations at

$$\begin{aligned} \sigma_R &= 3.0 \text{ yds.} \\ \sigma_B &= 0.01722 \text{ Deg.} \qquad (3.1) \\ \sigma_E &= 0.01722 \text{ Deg.} \end{aligned}$$

The values given in Equation (3.1) are those used by Parr [Ref. 4] and are used in this thesis in an attempt to

duplicate Parr's testing environment, and minimize procedural discrepancies.

The performance evaluation of the filters developed in Chapter II is based on their N-step prediction or position-state prediction accuracy. A detailed description of the method used in the determination of the error in the predicted position is given in Appendix C. The results presented in this chapter are the percentages of "hits" and "scares" computed over the entire ensemble. A "hit" is defined as an error in the predicted position of less than eight yards, while a "scare" is defined as an error in the predicted position of less than 16 yards, but greater than eight yards.

Initialization of each filter simulated was identical in all trials and is described in Chapter II by Equations (2.5), (2.8), and (2.9).

The elements of the covariance of the random forcing input matrix $\underset{\sim}{Q}(K)$ were determined by a two-dimensional optimization search. The assumption of linear uncoupled dynamics eliminates any spatial correlation, resulting in a diagonal $\underset{\sim}{Q}(K)$ matrix of the form

$$\underset{\sim}{Q}(K) = \begin{bmatrix} q_r & 0 & 0 \\ 0 & q_B & 0 \\ 0 & 0 & q_E \end{bmatrix} \tag{3.2}$$

If the system dynamics were truly linear, then the diagonal elements of (3.2) would represent the variances of the random forcing input to the system. However, since $\underset{\sim}{Q}(K)$ is also used to compensate for inaccuracies in the system model, q_r, q_B and q_E must be adjusted accordingly. To simplify the optimization process for both models, q_B and q_E were constrained to be equal throughout. The two-dimensional search optimized the filter's position-state prediction accuracy as a function of two variables, q_r and the pair $q_B = q_E$.

In the case of the constant-velocity model, the elements of the matrix $\underset{\sim}{Q}(K)$ serve as a means by which the bandwidth of the filter is increased to ensure that the filter will respond to target accelerations. This, of course, is in addition to compensation for the unmodeled nonlinearities of the system dynamics. Because of the dual role played by the matrix $\underset{\sim}{Q}(K)$, a brute force search was done to determine the elements of $\underset{\sim}{Q}(K)$ that gave the best position-state prediction performance. In a similar manner, for the filter based on the correlated random acceleration model, the bandwidth must be increased to permit sufficient response to targets exhibiting non-zero acceleration rates. Here again brute force search techniques were used to determine the $\underset{\sim}{Q}(K)$ matrix that gave the best position prediction performance.

For both models, once the optimum $\underset{\sim}{Q}(K)$ matrix was identified for a specific trajectory, that particular

filter configuration was tested against the remaining trajectories. The net result being that each optimized filter was tested against seven additional trajectories to ensure that the optimum $\underset{\sim}{Q}(K)$ had been found, and to determine the effect of using a $\underset{\sim}{Q}(K)$ selected for a track of one type against targets having different maneuvering characteristics.

B. SIMULATION RESULTS FOR THE CONSTANT-VELOCITY (L/S^2) MODEL

The results presented in this section are for the optimized filter configurations. The term "Filter 1" denotes the filter with the specific $\underset{\sim}{Q}(K)$ matrix that gave the best position prediction performance against trajectory 1. Thus, the simulation results for eight filters, or more precisely, eight filter configurations are presented.

As pointed out in the preceding section, the assumption of linear uncoupled dynamics results in the diagonal $\underset{\sim}{Q}(K)$ matrix given by Equation (3.2). Table I gives the diagonal elements of the $\underset{\sim}{Q}(K)$ matrix that provided the best position prediction performance for each test trajectory. The relative magnitude of the diagonal elements for the optimal filters are as expected. That is to say, the elements of $\underset{\sim}{Q}(K)$ for the non-maneuvering trajectories (1, 3, 7, 8) are lower in magnitude than those for the maneuvering trajectories (2, 4, 5, 6). Similarly, the elements of $\underset{\sim}{Q}(K)$ for the long-range maneuvering trajectories are lower in magnitude than those for the short-range maneuvering trajectories. Similar

Table I

DIAGONAL ELEMENTS OF THE OPTIMAL $\underset{\sim}{Q}(K)$ MATRICES
FOR THE CONSTANT-VELOCITY ($1/S^2$) MODEL

FILTER	q_{11}	q_{22}	q_{33}
1	1.00×10^{-6}	8.58×10^{-9}	8.58×10^{-9}
2	0.05	0.1153	0.1153
3	1.00×10^{-6}	1.13×10^{-10}	1.13×10^{-10}
4	0.025	0.006553	0.006553
5	0.0075	0.1372	0.1372
6	1.00×10^{-4}	5.55×10^{-7}	5.55×10^{-7}
7	0.01	3.37×10^{-7}	3.37×10^{-7}
8	0.005	1.66×10^{-7}	1.66×10^{-7}

observations may be made for the moderate level maneuvers as opposed to the evasive level maneuvers. Note that the elements of $\underset{\sim}{Q}(K)$ for the long-range evasive maneuver profile (trajectory 6) are smaller than those for the long-range moderate level maneuver profile (trajectory 4). This appears to be a reasonable phenomenon in light of the fact that overweighting the estimates for a faster more highly maneuvering target at long range could result in larger errors in the predicted position. It follows then that the bandwidth of the optimal filter for the long-range evasive maneuver should be somewhat smaller than the bandwidth for the moderate level maneuver at long range.

Table II gives the simulation results for the optimal filter configurations as well as for each optimal filter configuration against the other trajectories. The results shown are not surprising in that for a specific trajectory, the optimal filter configuration gave the best results. In a few isolated cases, other filter configurations gave results that approach those for the optimal configuration. Though this was not expected, the differences are not considered small enough to justify any additional effort in the optimization process.

Table III compares the hit/scare percentages from the simulations of the two models using the two different state vectors. The results for the spherical coordinate state vector represent the optimal performance against a specific

Table II

SIMULATION RESULTS FOR THE OPTIMIZED CONSTANT-VELOCITY MODEL FILTERS

FILTER	Trajectory 1		Trajectory 2		Trajectory 3		Trajectory 4	
	%hit	%scare	%hit	%scare	%hit	%scare	%hit	%scare
1	86.89	5.00	9.02	11.45	40.21	22.29	0.0	0.10
2	71.77	16.48	25.34	24.05	0.71	3.62	0.1	0.86
3	86.86	4.91	9.02	11.45	40.81	21.14	0.0	0.10
4	85.82	5.84	20.57	26.11	5.10	19.10	0.71	0.95
5	70.34	17.27	24.84	24.66	0.0	4.76	0.0	0.0
6	85.23	4.68	9.02	11.45	40.03	21.32	0.0	0.0
7	86.59	4.91	9.02	11.45	40.24	22.43	0.0	0.0
8	86.62	4.93	9.02	11.45	40.38	22.29	0.0	0.0

Table II

SIMULATION RESULTS FOR THE OPTIMIZED CONSTANT-VELOCITY MODEL FILTERS (Cont'd)

FILTER	Trajectory 5		Trajectory 6		Trajectory 7		Trajectory 8	
	%hit	%scare	%hit	%scare	%hit	%scare	%hit	%scare
1	12.17	16.05	7.00	0.14	85.22	5.35	69.10	11.36
2	27.62	11.64	0.46	1.57	61.80	20.17	3.57	14.29
3	12.43	15.79	7.00	0.14	86.07	5.35	69.09	11.36
4	28.95	12.40	0.04	0.11	82.70	8.62	27.29	37.25
5	29.33	12.17	0.0	0.0	59.87	20.72	4.25	12.71
6	12.17	16.02	8.13	0.11	85.14	5.36	68.42	11.27
7	11.95	16.24	6.96	0.0	86.22	5.20	69.05	11.39
8	11.69	16.52	7.00	0.0	86.14	5.17	69.20	12.21

Table III

COMPARISON OF RESULTS FOR THE SPHERICAL AND CARTESIAN COORDINATE STATE VECTOR CONSTANT-VELOCITY MODELS

	CONSTANT VELOCITY TRAJECTORIES				MANEUVERING TRAJECTORIES			
STATE VECTOR	SHORT-RANGE		LONG-RANGE		SHORT-RANGE		LONG-RANGE	
	%hit	%scare	%hit	%scare	%hit	%scare	%hit	%scare
SPHERICAL COORDINATE	86.89	5.00	40.81	21.14	25.34	24.05	0.71	0.95
CARTESIAN COORDINATE	88.4	3.00	34.7	24.7	15.20	13.30	0.33	1.10

trajectory. The results tabulated for the Cartesian coordinate state vector are the best of the results reported by Parr [Ref. 4].

Tables IV and V show the tracking performance of each optimal filter against its specific trajectory. For example, the tracking performance of filter 1 against trajectory 1 is given, and so on. The tabulated data represent the means of estimation error, defined by Equation (2.7), time averaged over the entire trajectory, and the RMS value of the deviation of the means of estimation error about their time average.

C. SIMULATION RESULTS FOR THE CORRELATED RANDOM ACCELERATION MODEL

The results presented in this section are for the optimized filter configurations. Since the optimization search for the ninth-order filter required a great deal more computer time, this model was only optimized against trajectories 1-3. As in the preceding section, the term "Filter 1A" denotes the filter with the specific $\underset{\sim}{Q}(K)$ matrix that gave the best position prediction performance against trajectory 1.

The tabulated data of Table VI gives the elements of the $\underset{\sim}{Q}(K)$ matrices that provided the optimum performance against each test trajectory. The data appear to be consistent within the tracking environments presented to the filter. Notice that in each configuration the elements of the optimum $\underset{\sim}{Q}(K)$ matrices for the correlated random

Table IV

POSITION ESTIMATION PERFORMANCE OF THE OPTIMIZED CONSTANT-VELOCITY MODEL FILTERS

FILTER	$\bar{e}_r$ (yds)	$\sigma_{\bar{e}_r}$ (yds)	$\bar{e}_\phi$ (deg)	$\sigma_{\bar{e}_\phi}$ (deg)	$\bar{e}_\theta$ (deg)	$\sigma_{\bar{e}_\theta}$ (deg)
1	5.33×10^{-2}	0.126	1.52×10^{-4}	6.18×10^{-4}	-2.67×10^{-4}	8.11×10^{-4}
2	-8.70×10^{-2}	0.282	2.32×10^{-3}	1.06×10^{-2}	2.90×10^{-6}	1.50×10^{-3}
3	4.95×10^{-2}	0.126	1.52×10^{-4}	6.18×10^{-4}	-2.68×10^{-4}	8.11×10^{-4}
4	-7.00×10^{-2}	0.219	1.69×10^{-3}	1.32×10^{-2}	5.22×10^{-6}	1.15×10^{-3}
5	9.04×10^{-2}	0.985	2.35×10^{-4}	1.95×10^{-2}	1.47×10^{-5}	1.56×10^{-3}
6	2.93×10^{-2}	0.894	-2.85×10^{-2}	2.93×10^{-1}	-3.77×10^{-4}	9.46×10^{-4}
7	6.00×10^{-2}	0.127	1.40×10^{-4}	6.21×10^{-4}	-2.50×10^{-4}	8.10×10^{-4}
8	5.25×10^{-2}	0.125	1.39×10^{-4}	6.19×10^{-4}	-2.60×10^{-4}	8.10×10^{-4}

Table V

RATE ESTIMATION PERFORMANCE OF THE OPTIMIZED CONSTANT-VELOCITY MODEL FILTERS

FILTER	$\bar{e}_{\dot{r}}$ (y/s)	$\sigma_{\bar{e}_{\dot{r}}}$ (y/s)	$\bar{e}_{\dot{\phi}}$ (d/s)	$\sigma_{\bar{e}_{\dot{\phi}}}$ (d/s)	$\bar{e}_{\dot{\theta}}$ (d/s)	$\sigma_{\bar{e}_{\dot{\theta}}}$ (d/s)
1	3.79	29.70	1.00×10^{-5}	5.26×10^{-4}	-2.45×10^{-4}	1.57×10^{-3}
2	3.69	29.72	3.36×10^{-2}	1.90×10^{-1}	-5.45×10^{-4}	5.71×10^{-3}
3	3.80	29.70	9.97×10^{-6}	5.26×10^{-4}	-2.45×10^{-4}	1.57×10^{-3}
4	3.70	29.71	3.75×10^{-3}	6.67×10^{-2}	-1.48×10^{-4}	2.18×10^{-3}
5	4.99	38.75	-2.51×10^{-2}	2.36×10^{-1}	-1.62×10^{-4}	5.77×10^{-3}
6	4.98	38.75	-1.15×10^{-2}	2.86×10^{-1}	-2.97×10^{-4}	1.68×10^{-3}
7	4.96	38.74	6.76×10^{-5}	5.27×10^{-4}	-2.38×10^{-4}	1.57×10^{-3}
8	4.97	38.73	6.67×10^{-5}	5.27×10^{-4}	-2.42×10^{-4}	1.57×10^{-3}

Table VI

DIAGONAL ELEMENTS OF THE OPTIMIZED $\underset{\sim}{Q}(K)$ MATRICES FOR THE CORRELATED RANDOM ACCELERATION MODEL FILTERS

FILTER	q_{11}	q_{22}	q_{33}
1A	1.00×10^{-7}	5.00×10^{-9}	5.00×10^{-9}
2A	0.100	14.599	14.599
3A	1.00×10^{-3}	1.46×10^{-7}	1.46×10^{-7}

acceleration model are larger in magnitude than those for the corresponding entries of the constant-velocity model. This seems reasonable because this filter is required to respond to a transient (due to the non-zero acceleration rate) that is expected to be smaller in magnitude than the transient encountered by the constant-velocity model filter. For this reason the bandwidth of this filter must be increased to ensure sufficient response to the non-zero acceleration rates. During the optimization search, it was observed that, with one exception, this filter model is extremely sensitive to variations of elements q_{22} and q_{33} corresponding to acceleration rates normal to the Line-of-Sight. This is to be expected, since such accelerations cause the greatest displacement from the nominal trajectory adding a source for potentially large errors in the predicted position. The single exception noted was for trajectory 1. In this specific case the filter was surprisingly insensitive to variations in the elements of the $\underset{\sim}{Q}(K)$ matrix, and its predicted position response was fairly flat over a four-decade range (10^{-11} - 10^{-7}). The elements reported for Filter 1A are therefore somewhat arbitrary, lying in the center of the flat region. Values for the pair $q_{22} = q_{33}$ above and below this region produced inferior results. A refined search grid in the flat region failed to produce any significant change in the results reported for Filter 1A.

Table VII gives the simulation results for each optimal filter configuration and for each optimal filter configuration against the two remaining trajectories. Table VIII compares the hit/scare percentages from the simulations of the models using the two different state vectors. The data presented for the spherical coordinate state vector represent the optimum performance for the trajectories considered. The results reported for the Cartesian Coordinate state vector model represent the best of the results reported by Parr [Ref. 4]. Tables IX through XI show the tracking performance for each optimal filter configuration against its specific trajectory. The results shown are the time-averaged means of estimation error and the RMS value of the deviation of the means of estimation error from their time average.

Table VII

SIMULATION RESULTS FOR THE OPTIMIZED CORRELATED RANDOM ACCELERATION MODELS

	TRAJECTORY 1		TRAJECTORY 2		TRAJECTORY 3	
FILTER	%hit	%scare	%hit	%scare	%hit	%scare
1A	74.41	7.27	7.20	8.27	8.95	19.48
2A	32.89	15.57	26.43	14.89	0.0	0.05
3A	74.09	7.55	7.16	8.27	12.93	18.52

Table VIII

COMPARISON OF RESULTS FOR THE SPHERICAL AND CARTESIAN COORDINATE STATE VECTOR CORRELATED RANDOM ACCELERATION MODELS

	CONSTANT-VELOCITY TRAJECTORIES				MANEUVERING TRAJECTORIES			
STATE VECTOR	SHORT-RANGE		LONG-RANGE		SHORT-RANGE		LONG-RANGE	
	%hit	%scare	%hit	%scare	%hit	%scare	%hit	%scare
SPHERICAL COORDINATE	74.41	7.27	12.93	18.52	26.43	14.89	NOT TESTED	
CARTESIAN COORDINATE	73.40	7.70	6.60	17.40	21.20	9.00	NOT TESTED	

Table IX

POSITION ESTIMATION PERFORMANCE OF THE OPTIMIZED CORRELATED RANDOM ACCELERATION MODEL FILTERS

	FILTER		
PARAMETER	1A	2A	3A
$\bar{e}_r$ (yds)	5.87×10^{-2}	0.111	1.06×10^{-2}
$\sigma_{\bar{e}_r}$ (yds)	0.125	0.289	0.135
$\bar{e}_\phi$ (deg)	-5.45×10^{-5}	6.59×10^{-5}	2.55×10^{-4}
$\sigma_{\bar{e}_\phi}$ (deg)	7.55×10^{-4}	2.37×10^{-3}	8.23×10^{-4}
$\bar{e}_\theta$ (deg)	1.48×10^{-4}	2.41×10^{-4}	1.94×10^{-4}
$\sigma_{\bar{e}_\theta}$ (deg)	7.43×10^{-4}	1.40×10^{-3}	7.34×10^{-4}

Table X

RATE ESTIMATION PERFORMANCE OF THE OPTIMIZED CORRELATED RANDOM ACCELERATION MODEL FILTERS

PARAMETER	FILTER 1A	 2A	 3A
$\bar{e}_{\dot{r}}$ (y/s)	3.77	3.83	3.78
$\sigma_{\bar{e}_{\dot{r}}}$ (y/s)	29.71	29.70	29.70
$\bar{e}_{\dot{\phi}}$ (d/s)	$2.20\text{x}10^{-4}$	$1.64\text{x}10^{-2}$	$5.06\text{x}10^{-3}$
$\sigma_{\bar{e}_{\dot{\phi}}}$ (d/s)	$1.76\text{x}10^{-3}$	0.109	$1.39\text{x}10^{-2}$
$\bar{e}_{\dot{\theta}}$ (d/s)	$-1.58\text{x}10^{-4}$	$-1.50\text{x}10^{-4}$	$1.63\text{x}10^{-3}$
$\sigma_{\bar{e}_{\dot{\theta}}}$ (d/s)	$3.31\text{x}10^{-3}$	$8.14\text{x}10^{-3}$	$2.67\text{x}10^{-3}$

Table XI

ACCELERATION ESTIMATION PERFORMANCE OF THE OPTIMIZED CORRELATED RANDOM ACCELERATION MODEL FILTERS

	FILTER		
PARAMETER	1A	2A	3A
$\bar{e}_{\ddot{r}}$ (y/s^2)	-0.546	-0.549	-0.526
$\sigma_{\bar{e}_{\ddot{r}}}$ (y/s^2)	3.32	3.39	3.17
$\bar{e}_{\ddot{\phi}}$ (d/s^2)	$-4.35\text{x}10^{-4}$	$1.82\text{x}10^{-2}$	$-1.12\text{x}10^{-3}$
$\sigma_{\bar{e}_{\ddot{\phi}}}$ (d/s^2)	$2.68\text{x}10^{-3}$	0.31	$1.39\text{x}10^{-2}$
$\bar{e}_{\ddot{\theta}}$ (d/s^2)	$-9.70\text{x}10^{-4}$	0.49	$5.39\text{x}10^{-4}$
$\sigma_{\bar{e}_{\ddot{\theta}}}$ (d/s^2)	$1.52\text{x}10^{-3}$	0.10	$2.67\text{x}10^{-3}$

IV. ADAPTIVE FILTERING

A. THE ADAPTIVE BANDWIDTH FILTER

A single fixed-parameter filter configuration cannot realistically be expected to perform well against all trajectory profiles likely to be encountered. One way of approaching this problem is to use a filter that adapts its characteristics to a changing tracking environment, e.g., a target maneuver is detected. The design problem now becomes two-fold; under what circumstances, or when, should the filter's characteristics be changed and by how much should the filter be changed? Price and Brown [Ref. 11] discuss some interesting approaches to the design of adaptive filters; unfortunately their complexity is beyond the defined scope of this thesis.

The single method to be considered here will be a simple switch-on-range scheme. The motivation for such a scheme lies in the simplicity of its implementation. Additionally, Parr reported [Ref. 4] that of the methods he investigated, the switch-on-range method provided the best overall performance in his simulations. When filter tracking is initiated, a low $\underset{\sim}{Q}(K)$ gain schedule (corresponding to a narrow bandwidth filter) is used to provide the optimum performance against long-range targets. When the target's measured range reaches a predetermined switching point, the filter switches to a high $\underset{\sim}{Q}(K)$ gain

schedule (corresponding to a wideband filter) for the short-range tracking situation.

Both filter models developed in Chapter II are simulated in a switch-on-range adaptive mode of operation. Again, to duplicate Parr's testing environment, a switching range of 3333.33 yards is used, and the filters are tested against trajectories 9 - 14. The profiles for trajectories 9 - 14 are identical to Parr's trajectories 1 - 6. The results reported for the adaptive simulations are the products of 80-member ensemble Monte Carlo simulations. For the simulations of the constant-velocity model, the low $\underset{\sim}{Q}(K)$ gain schedule used was that from Filter 3. The data in Table II show that Filter 3 gave the best overall performance against the long-range trajectories treated in Chapter III. Similar considerations yield Filter 4 as the source for the high $\underset{\sim}{Q}(K)$ gain schedule for the short-range tracking environment. When filter tracking is initiated, the Kalman gains used are those for Filter 3. When the target's measured range becomes less than or equal to the switching range of 3333.33 yards, the Kalman gain schedule is reset to the steady-state gain values obtained from the gain schedule for Filter 4. Similar consideration of the data presented in Table VII gives Filter 2A as the source of the high $\underset{\sim}{Q}(K)$ gain schedule while Filter 3A provides the low $\underset{\sim}{Q}(K)$ gain schedule. As in the above simulation, the low $\underset{\sim}{Q}(K)$ gain schedule is used until the

switching range (3333.33 yards) is reached. At that point, the Kalman gain schedule is reset to the steady-state gain values from gain schedule for Filter 2A. Initialization of the filters was the same for all simulations conducted and is described by Equations (2.5), (2.8), and (2.9). The statistics assumed in all simulations for the measurement noise process $\underset{\sim}{v}(K)$ are given by Equations (1.6) and (3.1).

B. SIMULATION RESULTS FOR THE CONSTANT-VELOCITY MODEL SWITCH-ON-RANGE ADAPTIVE FILTER

The constant-velocity model filter was tested in a switch-on-range adaptive mode of operation against trajectories 9 - 14. Table XII shows the filter's position prediction accuracy results. The data tabulated in Table XIII compare the results for the adaptive simulations of the models using the two different state vectors. Tables XIV and XV show the tracking performance of the filter in the adaptive mode. As in Chapter III the tabulated data represent the time-averaged means of estimation error and the RMS value of the deviation of the means of estimation error from their time average.

C. SIMULATION RESULTS FOR THE CORRELATED RANDOM ACCELERATION MODEL SWITCH-ON-RANGE ADAPTIVE FILTER

The correlated random acceleration model was also tested in an adaptive mode of operation against trajectories 9 - 14. Table XVI shows this filter's position prediction performance results. Tables XVII - XIX show the filter's

Table XII

SWITCH-ON-RANGE ADAPTIVE FILTER SIMULATION FOR THE CONSTANT-VELOCITY MODEL

TRAJECTORY	%HIT	%SCARE
9	90.58	5.55
10	60.57	12.98
11	71.58	11.39
12	63.95	15.45
13	67.37	9.98
14	74.67	9.65

Table XIII

COMPARISON OF RESULTS FOR THE SPHERICAL AND CARTESIAN COORDINATE STATE VECTOR CONSTANT-VELOCITY MODEL SWITCH-ON-RANGE ADAPTIVE FILTERS

	STATE VECTOR			
TRAJECTORY	SPHERICAL		CARTESIAN	
	%HIT	%SCARE	%HIT	%SCARE
9	90.58	5.55	81.80	11.60
10	60.57	12.98	47.20	10.60
11	71.58	11.39	60.10	14.20
12	63.95	15.45	52.30	13.40
13	67.37	9.98	60.20	15.00
14	74.67	9.67	54.70	13.10

Table XIV

POSITION ESTIMATION PERFORMANCE FOR THE CONSTANT-VELOCITY MODEL

SWITCH-ON-RANGE ADAPTIVE FILTER

TRAJECTORY	$\bar{e}_r$(yds)	$\sigma_{\bar{e}_r}$ (yds)	$\bar{e}_\phi$(deg)	$\sigma_{\bar{e}_\phi}$ (deg)	$\bar{e}_\theta$(deg)	$\sigma_{\bar{e}_\theta}$ (deg)
9	6.34×10^{-2}	0.105	1.31×10^{-4}	8.59×10^{-4}	-1.18×10^{-5}	9.01×10^{-4}
10	-2.82×10^{-3}	0.261	1.52×10^{-2}	6.52×10^{-2}	3.59×10^{-6}	9.16×10^{-4}
11	-3.30×10^{-2}	0.228	4.60×10^{-3}	3.46×10^{-2}	-4.79×10^{-5}	1.03×10^{-3}
12	-1.30×10^{-2}	0.220	1.12×10^{-4}	2.30×10^{-2}	-1.18×10^{-5}	9.30×10^{-4}
13	-0.131	0.478	1.14×10^{-4}	4.33×10^{-2}	-1.18×10^{-5}	9.75×10^{-4}
14	-0.167	0.509	6.14×10^{-3}	7.85×10^{-2}	-1.2×10^{-4}	1.75×10^{-3}

Table XV

RATE ESTIMATION PERFORMANCE FOR THE CONSTANT-VELOCITY MODEL SWITCH-ON-RANGE ADAPTIVE FILTER

TRAJECTORY	$\bar{e}_{\dot{r}}$ (y/s)	$\sigma_{\bar{e}_{\dot{r}}}$ (y/s)	$\bar{e}_{\dot{\phi}}$ (d/s)	$\sigma_{\bar{e}_{\dot{\phi}}}$ (d/s)	$\bar{e}_{\dot{\theta}}$ (d/s)	$\sigma_{\bar{e}_{\dot{\theta}}}$ (d/s)
9	1.27	15.81	6.94×10^{-5}	1.16×10^{-3}	-1.14×10^{-4}	1.50×10^{-3}
10	1.26	15.81	1.75×10^{-3}	9.92×10^{-2}	-1.46×10^{-4}	1.93×10^{-3}
11	1.24	15.81	1.96×10^{-2}	0.165	-1.24×10^{-3}	6.84×10^{-3}
12	1.25	15.81	-1.97×10^{-3}	0.109	-2.28×10^{-4}	2.40×10^{-3}
13	1.22	15.82	-6.48×10^{-3}	0.216	-1.87×10^{-4}	3.71×10^{-3}
14	1.18	15.82	1.47×10^{-2}	0.453	-1.92×10^{-3}	1.56×10^{-2}

tracking performance in the adaptive mode. As in the preceding section, the tabulated data represent the time-averaged means of estimation error and the RMS value of the deviation of the means of estimation error from their time average.

Table XVI

SWITCH-ON-RANGE ADAPTIVE FILTER SIMULATION FOR THE CORRELATED RANDOM ACCELERATION MODEL

TRAJECTORY	%hit	%scare
9	62.27	10.85
10	46.72	15.20
11	60.46	10.76
12	53.34	13.06
13	56.67	9.36
14	56.08	11.23

Table XVII

POSITION ESTIMATION PERFORMANCE FOR THE CORRELATED RANDOM ACCELERATION SWITCH-ON-RANGE ADAPTIVE FILTER

TRAJECTORY	$\bar{e}_r$(yds)	$\sigma_{\bar{e}_r}$ (yds)	$\bar{e}_\phi$(deg)	$\sigma_{\bar{e}_\phi}$ (deg)	$\bar{e}_\theta$(deg)	$\sigma_{\bar{e}_\theta}$ (deg)
9	$3.64\text{x}10^{-2}$	0.12	$1.57\text{x}10^{-4}$	$1.13\text{x}10^{-3}$	$1.81\text{x}10^{-4}$	$1.11\text{x}10^{-3}$
10	$1.01\text{x}10^{-2}$	0.20	$-1.42\text{x}10^{-2}$	$5.69\text{x}10^{-2}$	$1.96\text{x}10^{-4}$	$1.12\text{x}10^{-3}$
11	$-1.24\text{x}10^{-3}$	0.16	$1.66\text{x}10^{-4}$	$1.49\text{x}10^{-3}$	$1.82\text{x}10^{-4}$	$1.11\text{x}10^{-3}$
12	$1.57\text{x}10^{-2}$	0.17	$1.60\text{x}10^{-4}$	$1.84\text{x}10^{-3}$	$1.81\text{x}10^{-4}$	$1.11\text{x}10^{-3}$
13	$-2.10\text{x}10^{-2}$	0.31	$1.57\text{x}10^{-4}$	$3.98\text{x}10^{-3}$	$1.81\text{x}10^{-4}$	$1.11\text{x}10^{-3}$
14	$-7.32\text{x}10^{-2}$	0.32	$-1.58\text{x}10^{-5}$	$5.01\text{x}10^{-3}$	$1.85\text{x}10^{-4}$	$1.13\text{x}10^{-3}$

Table XVIII

RATE ESTIMATION PERFORMANCE FOR THE CORRELATED RANDOM ACCELERATION MODEL SWITCH-ON-RANGE ADAPTIVE FILTER

TRAJECTORY	$\bar{e}_{\dot{r}}$ (y/s)	$\sigma_{\bar{e}_{\dot{r}}}$ (y/s)	$\bar{e}_{\dot{\phi}}$ (d/s)	$\sigma_{\bar{e}_{\dot{\phi}}}$ (d/s)	$\bar{e}_{\dot{\theta}}$ (d/s)	$\sigma_{\bar{e}_{\dot{\theta}}}$ (d/s)
9	1.26	15.81	2.16×10^{-4}	4.95×10^{-3}	2.87×10^{-4}	5.16×10^{-3}
10	1.26	15.82	2.70×10^{-3}	0.11	2.56×10^{-4}	5.32×10^{-3}
11	1.24	15.82	2.76×10^{-3}	4.44×10^{-2}	-7.30×10^{-4}	7.87×10^{-3}
12	1.25	15.82	-1.82×10^{-3}	2.03×10^{-2}	1.73×10^{-4}	5.36×10^{-3}
13	1.24	15.82	-6.35×10^{-3}	5.70×10^{-2}	2.14×10^{-4}	5.79×10^{-3}
14	1.20	15.84	-9.02×10^{-3}	0.20	-1.15×10^{-3}	1.15×10^{-2}

Table XIX

ACCELERATION ESTIMATION PERFORMANCE FOR THE CORRELATED RANDOM ACCELERATION MODEL SWITCH-ON-RANGE ADAPTIVE FILTER

TRAJECTORY	$\bar{e}_{\ddot{r}}$ (y/s^2)	$\sigma_{\bar{e}_{\ddot{r}}}$ (y/s^2)	$\bar{e}_{\ddot{\phi}}$ (d/s^2)	$\sigma_{\bar{e}_{\ddot{\phi}}}$ (d/s^2)	$\bar{e}_{\ddot{\theta}}$ (d/s^2)	$\sigma_{\bar{e}_{\ddot{\theta}}}$ (d/s^2)
9	-1.09	1.72	$3.70\text{x}10^{-4}$	$1.23\text{x}10^{-2}$	$-4.94\text{x}10^{-2}$	$3.99\text{x}10^{-2}$
10	-0.112	1.73	$-3.50\text{x}10^{-3}$	0.236	$-4.84\text{x}10^{-3}$	2.76
11	0.108	1.73	$-3.07\text{x}10^{-2}$	0.566	-0.266	3.37
12	-0.112	1.73	$-1.01\text{x}10^{-2}$	0.111	$-1.44\text{x}10^{-2}$	2.99
13	-0.119	1.81	$-1.98\text{x}10^{-2}$	0.263	$7.91\text{x}10^{-2}$	6.34

V. SUMMARY AND CONCLUSIONS

A. SUMMARY OF TEST RESULTS

1. Summary of Simulation Results for the Two Models

The results presented in Chapter III show that against non-maneuvering target profiles the optimized constant-velocity models provided superior prediction performance. When the bandwidth of the filter is increased, adequate position prediction performance against maneuvering targets is obtained but at the expense of performance degradation against the non-maneuvering profiles. The optimized correlated random acceleration model showed only a slight improvement against the maneuvering profiles considered, but produced results inferior to those obtained from the constant-velocity model against the non-maneuvering target profiles. The performance of the constant-velocity model against long-range maneuvering profiles was significantly lower than for the short-range maneuvering profiles. This is to be expected since the long-range profiles require that the predictions be made over a longer period of time during which the target may maneuver unexpectedly and significantly. The tabulated data presenting the tracking performance of the filters developed in Chapter II, show that the position estimation performance for both filters is approximately the same. The correlated random acceleration model gave only

slightly better rate estimation performance than did the constant-velocity model. Against the target profiles considered the filter models derived from a spherical coordinate state vector provided significantly better overall predicted position results than the filter models derived from a Cartesian Coordinate state vector.

2. Summary of Simulation Results for the Adaptive Filters

The results reported in Chapter IV for the simulation of the switch-on-range adaptive filters indicated that the constant-velocity model derived from a spherical coordinate state vector provided significantly better predicted position performance than the constant-velocity model filter derived from a Cartesian Coordinate state vector against all target profiles considered. Further, the results indicate that of the models derived from a spherical coordinate state vector, the constant-velocity model gave the best overall predicted position performance.

B. CONCLUSIONS

A filter derived from a spherical coordinate state vector with assumed linear system dynamics produces predicted position results potentially superior to those obtained from a filter derived from a Cartesian Coordinate state vector. The spherical coordinate state vector constant-velocity model provides better overall predicted

position results than the correlated random acceleration model derived from a spherical coordinate state vector. The reason for this is perhaps due to the manner in which the predicted position, $\hat{\underset{\sim}{x}}(K+N/K)$ is generated. For both models considered, the predicted position $\hat{\underset{\sim}{x}}(K+N/K)$ is formed by raising the state transition matrix $\underset{\sim}{\phi}(K,K-1)$, to be the $N^{\underline{th}}$ power as shown by Equation (C.2). In the case of the constant-velocity model such a method uses the current position and velocity estimates to generate the predicted position $\hat{\underset{\sim}{x}}(K+N/K)$. The correlated random acceleration model uses the current position, velocity and acceleration estimates to generate the predicted position. Thus, any error in the estimate of the target's acceleration state causes significant errors in the predicted position when propagated over the prediction interval. These acceleration induced prediction errors are not present when only the current estimates of position and velocity are used as in the case of the constant-velocity model.

A filter model derived from a spherical coordinate state vector can produce results potentially superior to those obtained from the corresponding Cartesian Coordinate state vector model in an adaptive filtering environment.

C. SUGGESTIONS FOR FURTHER INVESTIGATION

Since the constant-velocity model produced the best overall predicted position results, a complete analysis of the sensitivity of this model's predicted position

performance to the statistics assumed for the measurement noise process $\underset{\sim}{v}(K)$ and the sampling period T should be carried out. Additional sensitivity studies should also focus on allowing the diagonal elements of $\underset{\sim}{Q}(K)$ to vary independently, thus removing the constraint that $q_{22} = q_{33}$.

For the correlated random acceleration model, one area that warrants future investigation is the use of only the position and velocity estimates of the filter in generating the predicted position, and comparing the results with those obtained from the constant-velocity model filter. This would reduce the errors in the predicted position due to errors in the current estimate of the target's acceleration state.

Both models should be tested against a greater variety of target profiles, especially targets that present crossing profiles in which large bearing and elevation rates are generated, and targets that execute maneuvers while in level flight.

Finally, more sophisticated adaptive filtering schemes should be developed and investigated for the constant-velocity model. A prime candidate for such an investigation is the residual testing method reported by Price and Brown [Ref. 11].

APPENDIX A

MONTE CARLO SIMULATION PROGRAM

A.1 PROGRAM LISTING AND DESCRIPTION

The Monte Carlo Simulation Program used in this thesis was written by Dr. D. E. Kirk of the Naval Postgraduate School. It is extremely flexible and well documented to assist the potential user in data preparation and input, as well as the various options available. It should be pointed out that SUBROUTINE KILL and all statements in the main program that deal with miss distance data were written specifically for this thesis. They may be deleted with no loss of program flexibility or functional operation.

Computer system requirements vary with the simulation and the user's desires. All the computer work done for this thesis was performed on the IBM SYSTEM 360/67 at the Naval Postgraduate School. The program as currently dimensioned requires 520K bytes of storage. When redimensioned for the 60 data point trajectories, 320K bytes of storage are required. A 100-member ensemble simulation of a linear sixth-order filter with no on-line computation required, the trajectory read in, miss distance information desired, and all printing and plotting options invoked requires approximately 1.5 minutes of CPU time for execution. An additional 30 seconds (approximately) are required for the compile/link process. For the work done for this thesis

the program was pre-compiled (FORTRAN IV Level G) and stored and executed from a data cell. The link process in this case required approximately two seconds. If the same linear, sixth-order filter is again simulated but the printed and plotted output is suppressed (retaining only the miss distance data) 59 seconds of CPU time is required to execute the simulation. In the case of the linear ninth-order filters that were simulated, the storage requirements remain unchanged, as do the time requirements for the compile/link process. A 100-member ensemble simulation of the linear ninth-order filter with no on-line computation required, miss distance information desired, the trajectory read in and all printing and plotting options invoked requires approximately 3.0 minutes of CPU time for execution. If only the miss distance information is desired and all other output is suppressed, approximately 2.35 minutes of CPU time is required to execute the simulation.

```
C     THIS PRCGRAM PERFORMS MONTE CARLO SIMULATION OF STATE ESTIMATORS  MCSP   1
C     OF WHICH THE KALMAN FILTER IS ONE EXAMPLE. THERE ARE SEVERAL      MCSP   2
C     CPTICNS AVAILABLE AS INDICATED IN THE DETAILLED COMMENTS BELOW.   MCSP   3
C     IT SHOULD BE NOTED THAT ALL COMPUTATIONS OF GAINS USING THE       MCSP   4
C     SUBROUTINE GAIN ARE PERFORMED IN DOUBLE PRECISION. THUS ALL       MCSP   5
C     AFRAYS FOR USE IN "GAIN" MUST BE PREPARED ACCORDINGLY.            MCSP   6
C                                                                       MCSP   7
      REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI,PKKM10MCSP   8
     1,MISSD                                                            MCSP   9
      CCMMON GAMMA(9,9),COVW(9,9),R(9,9),PHI(9,9),H(9,9),TEMP(9,9),TEMP1MCSP  10
     1(9,9),TEMP2(9,9),PKKM1(9,9),PKKM10(9,9),G(9,9),PKK(9,9),Q(9,9),EI(MCSP  11
     29,9),MISSD(100,160),VAR(9,9,160),GKS(9,9,160),PKKS(9,9,160),XM(9,1MCSP  12
     360),ERR(9,160),GAMMAS(9,9),PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGMCSP  13
     4W(9),X(9),SIGXZ(9),XZMEAN(9),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SMCSP  14
     5IGV(9),XHATZ(9),XINT(160),N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXMCSP  15
     6Z,IV,IW,IEST,ND,NHIT,NSCARE,NMISS,NFIRE                           MCSP  16
      DIMENSION XP(160), YP(160), SMISS(100,160), TEMPA(160)            MCSP  17
C                                                                       MCSP  18
C                                                                       MCSP  19
C     N=ORDER OF SYSTEM MODEL AND FILTER (DIMENSICN OF X,XHAT)          MCSP  20
C                                                                       MCSP  21
C     M=NUMBER CF MEASUREMENTS (DIMENSION OF THE VECTOR Z)              MCSP  22
C                                                                       MCSP  23
C     IN=NUMBER OF INPUT RANDOM FORCING FCNS (=DIMENSION OF W)          MCSP  24
C                                                                       MCSP  25
C     NSAM=NUMBER OF TIME SAMPLES                                       MCSP  26
C                                                                       MCSP  27
C     NENS=NUMBER OF MEMBERS IN ENSEMBLE                                MCSP  28
C                                                                       MCSP  29
      READ (5,106) N,M,IN,NSAM,NENS                                     MCSP  30
C                                                                       MCSP  31
C                                                                       MCSP  32
      READ (5,107) ND                                                   MCSP  33
C     THE VALUE OF ND READ IN MUST EQUAL THE ROW (AND COLUMN) DIMENSION MCSP  34
C     SPECIFIED FOR THE SQUARE MATRIX "TEMP1", E.G. IF TEMP1(3,3) IS    MCSP  35
C     SPECIFIED IN THE COMMON STATEMENT "ND" MUST BE EQUAL TO 3.        MCSP  36
C                                                                       MCSP  37
C                                                                       MCSP  38
C     IG=-1 -- GAINS COMPUTED OFF-LINE AND READ IN                      MCSP  39
C         0 -- GAINS COMPUTED ONLY ONCE BEFORE STARTING MONTE CARLO     MCSP  40
C         1 -- GAINS COMPUTED FOR EACH MEMBER OF ENSEMBLE               MCSP  41
C                                                                       MCSP  42
C     IFLR=0 -- R IS READ IN                                            MCSP  43
C     IFLR.NE.0 -- R IS COMPUTED ON-LINE AT EACH TIME SAMPLE            MCSP  44
C                                                                       MCSP  45
C     IEST=0 -- STANDARD KALMAN FILTER EQUATION IS USED                 MCSP  46
C     IEST.NE.0 -- STD. KALMAN FILTER EQ. NOT USED                      MCSP  47
C                                                                       MCSP  48
```

```
C     ITRK= 0 -- SEVERAL TRACKS GENERATED FROM STD LINEAR EQS           MCSP  49
C     ITRK= 1 -- ONLY ONE TRACK IS USED                                 MCSP  50
C     ITRK NE 0 OR 1 -- SEVERAL TRACKS GENERATED BUT NOT FROM STANDARD  MCSP  51
C                       LINEAR DIFFERENCE EQUATIONS.                    MCSP  52
C                                                                       MCSP  53
C     ISTAT=0 -- MEAN OF TRACK,MEAN & VARIANCE OF EST. ERROR COMPUTED   MCSP  54
C     ISTAT.NE.0 -- SAME AS ISTAT=0 BUT OFF-DIAGONAL TERMS IN COVARIANCEMCSP  55
C                                                                       MCSP  56
C                   MATRIX ARE ALSO COMPUTED                            MCSP  57
C                                                                       MCSP  58
      READ (5,108) IG,IFLR,IEST,ITRK,ISTAT,IQ,ITRC                      MCSP  59
C                                                                       MCSP  60
C     IQ=-1 -- Q IS COMPUTED ON-LINE AT EACH SAMPLE BY "QON"            MCSP  61
C         0 -- Q MATRIX IS READ IN                                      MCSP  62
C         1 -- COVARIANCE OF W READ IN AND Q IS  COMPUTED ONCE BEFORE   MCSP  63
C              MONTE CARLO BEGINS BY CALLING "QMAT"                     MCSP  64
C                                                                       MCSP  65
C     ITRO=0 -- ONLY USED IF ONE TRACK; ITRO=0 INDICATES THE            MCSP  66
C               TRACK IS READ IN (S.P.), OTHERWISE THE TRACK            MCSP  67
C               IS GENERATED BY A SINGLE CALL OF SUBROUTINE             MCSP  68
C               TRACK.                                                  MCSP  69
C                                                                       MCSP  70
C     IPRT=0 -- SOME OR ALL OUTPUT DATA IS PRINTED                      MCSP  71
C                                                                       MCSP  72
C     IPLT=0 -- SOME OR ALL OUTPUT DATA IS PLOTTED                      MCSP  73
      READ (5,109) IPRT,IPLT                                            MCSP  74
C                                                                       MCSP  75
C     IGPLT,ITHVPL,IMTPLT,ISMPLT,ISVPLT ARE PLOTTING INDICATORS         MCSP  76
C     IF THE INDICATOR IS EQUAL TO 1 PLOTS ARE GENERATED. THE INDICATORSMCSP  77
C     CORRESPOND TO THE FOLLOWING QUANTITIES                            MCSP  78
C     IGPLT(GAIN VS K),ITHVPL (THEORETICAL COVARIANCE MATRIX VS K),     MCSP  79
C     IMTPLT (SAMPLE MEAN OF TRACK=TRACK IF ONLY ONE TRACK),            MCSP  80
C     ISMPLT (SAMPLE MEAN OF EST. ERROR VS K), ISVPLT (SAMPLE VARIANCE  MCSP  81
C     OF EST. ERROR VS K)                                               MCSP  82
      IF (IPLT.EQ.0) READ (5,110) IGPLT,ITHVPL,IMTPLT,ISMPLT,ISVPLT     MCSP  83
C                                                                       MCSP  84
C     IPHI=0 -- PHI MATRIX TO BE READ IN (IN DOUBLE PREC)               MCSP  85
C     IPHI.NE.0 -- PHI MATRIX NOT TO BE READ                            MCSP  86
C                                                                       MCSP  87
C     IH=0 -- H MATRIX TO BE READ IN (IN D.P.)                          MCSP  88
C                                                                       MCSP  89
C     IR=0 -- R MATRIX IS READ IN                                       MCSP  90
C                                                                       MCSP  91
C     IPKKMO=0 -- THE STARTING VALUE FOR THE MATRIX P(K/K-1) IS TO      MCSP  92
C                 BE READ IN (D.P.)                                     MCSP  93
C                                                                       MCSP  94
C     IGAM=0 -- GAMMA MATRIX TO BE READ IN (D.P.)                       MCSP  95
C                                                                       MCSP  96
```

```
C     ISIGV=0 -- STD. DEVIATIONS OF MEASUREMENT NCISE TO BE READ IN (SP)MCSP  97
C                                                                       MCSP  98
C     ISIGW=0 -- STD. DEVIATIONS OF RANCCM INPUTS TO BE READ IN (SP)    MCSP  99
C                                                                       MCSP 100
C     IXHZ=0 -- INITIAL VALUE OF THE PREDICTED VALUE XH(0/-1) TO        MCSP 101
C               BE READ IN (S.P.)                                       MCSP 102
C                                                                       MCSP 103
C     IC=1 -- ONE INITIAL CONDITION VALUE USED FOR THE STATE            MCSP 104
C             OTHERWISE THE VALUE OF X(0) IS GENERATED BY A RANDOM      MCSP 105
C             NUMBER GENERATOR CALLED BY "XZERC".                       MCSP 106
C                                                                       MCSP 107
      READ (5,111) IPHI,IH,IR,IPKKMO,IGAM,ISIGV,ISIGW,IXHZ,IC,IMISS,IFIRMCSP 108
     1E,IMISSK,IMISSR,IMISSI,IMISSC                                     MCSP 109
C                                                                       MCSP 110
C     IMISS=0       FIRING AND MISS DISTANCE DATA ARE DESIRED.  OTHER-  MCSP 111
C                   WISE NOT.                                           MCSP 112
C                                                                       MCSP 113
C     IFIRE=0       FIRING DATA AVERAGEC OVER ENTIRE MONTE CARLO RUN IS MCSP 114
C                   PRINTED.  OTHERWISE NOT.                            MCSP 115
C                                                                       MCSP 116
C     IMISSK=0      MEAN AND STANDARD DEVIATION OF THE MISS DISTANCE AREMCSP 117
C                   PLOTTED VS. K.  OTHERWISE NCT.                      MCSP 118
C                                                                       MCSP 119
C     IMISSR=0      MEAN AND STANDARD DEVIATION OF THE MISS DISTANCE AREMCSP 120
C                   PLOTTED VS. THE TARGET SLANT RANGE.  OTHERWISE NOT. MCSP 121
C                                                                       MCSP 122
C     IMISSI=0      MEAN AND STANDARD DEVIATION OF THE MISS DISTANCE AREMCSP 123
C                   PLOTTED VS. THE AVERAGE LENGTH OF THE PREDICTION    MCSP 124
C                   INTERVAL.  OTHERWISE NOT.                           MCSP 125
C                                                                       MCSP 126
C     IMISSC=0      MEANS AND STANDARD DEVIATIONS OF THE MISS DISTANCE  MCSP 127
C                   ARE PRINTED OUT.  OTHERWISE NCT.                    MCSP 128
C                                                                       MCSP 129
      NHIT = 0                                                          MCSP 130
      NMISS = 0                                                         MCSP 131
      NSCARE = 0                                                        MCSP 132
      IRCS = 0                                                          MCSP 133
      NFIRE = 0                                                         MCSP 134
      IMISSN = 0                                                        MCSP 135
      ISCARE = 0                                                        MCSP 136
      CALL CVFLOW                                                       MCSP 137
      IW = 6395217                                                      MCSP 138
      IV = 1936748                                                      MCSP 139
      IXZ = 135769                                                      MCSP 140
C                                                                       MCSP 141
C                                                                       MCSP 142
C     THE FOLLOWING SECTION PRINTS OUT A DESCRIPTICN OF THE RUN AS      MCSP 143
C     SPECIFIED BY THE USER'S FLAGS                                     MCSP 144
```

```
C                                                                       MCSP 145
C                                                                       MCSP 146
      WRITE (6,181)                                                     MCSP 147
      WRITE (6,112)                                                     MCSP 148
      IF (IG.EQ.0) GO TO 1                                              MCSP 149
      IF (IG.EQ.1) GO TO 2                                              MCSP 150
      WRITE (6,113)                                                     MCSP 151
      GC TO 3                                                           MCSP 152
    1 WRITE (6,114)                                                     MCSP 153
      GC TO 3                                                           MCSP 154
    2 WRITE (6,115)                                                     MCSP 155
    3 IF (IEST.EQ.0) GO TO 4                                            MCSP 156
      WRITE (6,116)                                                     MCSP 157
      GC TO 5                                                           MCSP 158
    4 WRITE (6,117)                                                     MCSP 159
    5 IF (ITRK.EQ.0) GO TO 8                                            MCSP 160
      IF (ITRK.EQ.-1) GO TO 7                                           MCSP 161
      IF (ITRO.EQ.0) GO TO 6                                            MCSP 162
      WRITE (6,118)                                                     MCSP 163
      GC TO 9                                                           MCSP 164
    6 WRITE (6,119)                                                     MCSP 165
      GC TO 9                                                           MCSP 166
    7 WRITE (6,120)                                                     MCSP 167
      GC TO 9                                                           MCSP 168
    8 WRITE (6,121)                                                     MCSP 169
    9 IF (ISTAT.EQ.0) GO TO 10                                          MCSP 170
      WRITE (6,122)                                                     MCSP 171
      GC TO 11                                                          MCSP 172
   10 WRITE (6,123)                                                     MCSP 173
   11 IF (IQ.EQ.0) GO TO 13                                             MCSP 174
      IF (IQ.EQ.1) GO TO 12                                             MCSP 175
      WRITE (6,124)                                                     MCSP 176
      GC TO 14                                                          MCSP 177
   12 WRITE (6,125)                                                     MCSP 178
      GC TO 14                                                          MCSP 179
   13 WRITE (6,126)                                                     MCSP 180
   14 IF (IFLR.EQ.0) GO TO 15                                           MCSP 181
      WRITE (6,127)                                                     MCSP 182
      GC TO 16                                                          MCSP 183
   15 WRITE (6,128)                                                     MCSP 184
   16 WRITE (6,129)                                                     MCSP 185
      IF (IPHI.EQ.0) WRITE (6,130)                                      MCSP 186
      IF (IH.EQ.0) WRITE (6,131)                                        MCSP 187
      IF (IFLR.EQ.0) WRITE (6,132)                                      MCSP 188
      IF (IQ.EQ.0) GO TO 17                                             MCSP 189
      IF (IQ.EQ.-1) GO TO 18                                            MCSP 190
      WRITE (6,133)                                                     MCSP 191
      GC TO 18                                                          MCSP 192
```

```
   17 WRITE (6,134)                                                       MCSP 193
   18 IF (IGAM.EQ.0) WRITE (6,135)                                        MCSP 194
      IF (ISIGV.EQ.0) WRITE (6,136)                                       MCSP 195
      IF (ISIGW.EQ.0) WRITE (6,137)                                       MCSP 196
      IF (IXHZ.EQ.0) WRITE (6,138)                                        MCSP 197
      IF (IC.NE.1) WRITE (6,139)                                          MCSP 198
      IF (IPKKM1.EQ.0) WRITE (6,140)                                      MCSP 199
C                                                                         MCSP 200
C                                                                         MCSP 201
C     TFE FOLLOWING SECTION PRINTS OUT A DESCRIPTICN OF THE OUTPUT        MCSP 202
C     DATA CALLED FOK                                                     MCSP 203
C                                                                         MCSP 204
C                                                                         MCSP 205
      WRITE (6,141)                                                       MCSP 206
      IF (IPRT.NE.0) GO TO 19                                             MCSP 207
      WRITE (6,142)                                                       MCSP 208
      IF (IG.EQ.0) WRITE (6,143)                                          MCSP 209
      WRITE (6,144)                                                       MCSP 210
      IF (ISTAT.NE.0) WRITE (6,145)                                       MCSP 211
      GC TO 20                                                            MCSP 212
   19 WRITE (6,146)                                                       MCSP 213
   20 IF (IMISS.NE.0) GO TO 21                                            MCSP 214
      IF (IFIRE.EQ.0) WRITE (6,192)                                       MCSP 215
      IF (IMISSC.EQ.0) WRITE (6,193)                                      MCSP 216
      GC TO 22                                                            MCSP 217
   21 WRITE (6,194)                                                       MCSP 218
   22 IF (IPLT.NE.0) GO TO 23                                             MCSP 219
      WRITE (6,147)                                                       MCSP 220
      IF (IGPLT.EQ.1) WRITE (6,148)                                       MCSP 221
      IF (ITHVPL.EQ.1) WRITE (6,149)                                      MCSP 222
      IF (IMTPLT.EQ.1) WRITE (6,150)                                      MCSP 223
      IF (ISMPLT.EQ.1) WRITE (6,151)                                      MCSP 224
      IF (ISVPLT.EQ.1) WRITE (6,152)                                      MCSP 225
      GC TO 24                                                            MCSP 226
   23 WRITE (6,153)                                                       MCSP 227
   24 CCNTINUE                                                            MCSP 228
      IF (IMISS.NE.0) GO TO 25                                            MCSP 229
      WRITE (6,208)                                                       MCSP 230
      IF (IMISSK.EQ.0) WRITE (6,189)                                      MCSP 231
      IF (IMISSR.EQ.0) WRITE (6,190)                                      MCSP 232
      IF (IMISSI.EQ.0) WRITE (6,191)                                      MCSP 233
      GC TO 26                                                            MCSP 234
   25 WRITE (6,209)                                                       MCSP 235
   26 CONTINUE                                                            MCSP 236
      WRITE (6,181)                                                       MCSP 237
      WRITE (6,154)                                                       MCSP 238
      WRITE (6,155) N,M,IN,NSAM,NENS,ND                                   MCSP 239
C                                                                         MCSP 240
```

```
C                                                                        MCSP 241
C     THE FOLLOWING SECTION READS THE SPECIFIED INPUT MATRICES          MCSP 242
C                                                                        MCSP 243
      IF (IPHI.NE.0) GO TO 28                                            MCSP 244
      CALL MREAD (PHI,N,N)                                               MCSP 245
C                                                                        MCSP 246
      DC 27 I=1,N                                                        MCSP 247
C                                                                        MCSP 248
      DC 27 J=1,N                                                        MCSP 249
   27 PHIS(I,J) = PHI(I,J)                                               MCSP 250
C                                                                        MCSP 251
      WRITE (6,156)                                                      MCSP 252
      CALL MWRITE (PHI,N,N)                                              MCSP 253
C                                                                        MCSP 254
C                                                                        MCSP 255
   28 IF (IH.NE.0) GO TO 30                                              MCSP 256
      CALL MREAD (H,M,N)                                                 MCSP 257
C                                                                        MCSP 258
      DO 29 I=1,M                                                        MCSP 259
C                                                                        MCSP 260
      DC 29 J=1,N                                                        MCSP 261
   29 HS(I,J) = H(I,J)                                                   MCSP 262
C                                                                        MCSP 263
      WRITE (6,157)                                                      MCSP 264
      CALL MWRITE (H,M,N)                                                MCSP 265
C                                                                        MCSP 266
C                                                                        MCSP 267
   30 IF (IR.NE.0) GO TO 31                                              MCSP 268
      CALL MREAD (R,M,M)                                                 MCSP 269
      WRITE (6,158)                                                      MCSP 270
      CALL MWRITE (R,M,M)                                                MCSP 271
C                                                                        MCSP 272
C                                                                        MCSP 273
   31 IF (IQ.NE.1) GO TO 32                                              MCSP 274
      CALL MREAD (COVW,IN,IN)                                            MCSP 275
      WRITE (6,159)                                                      MCSP 276
      CALL MWRITE (COVW,IN,IN)                                           MCSP 277
      GC TO 33                                                           MCSP 278
   32 IF (IQ.NE.0) GO TO 33                                              MCSP 279
      CALL MREAD (Q,N,N)                                                 MCSP 280
      WRITE (6,160)                                                      MCSP 281
      CALL MWRITE (Q,N,N)                                                MCSP 282
C                                                                        MCSP 283
C                                                                        MCSP 284
   33 IF (IGAM.NE.0) GO TO 35                                            MCSP 285
      CALL MREAD (GAMMA,N,IN)                                            MCSP 286
C                                                                        MCSP 287
      DC 34 I=1,N                                                        MCSP 288
```

```
C
      DC 34 J=1,IN
   34 GAMMAS(I,J) = GAMMA(I,J)
C
C
C
      WRITE (6,161)
      CALL MWRITE (GAMMA,N,IN)
C
C
   35 IF (IPKKMO.NE.0) GO TO 36
      CALL MREAD (PKKM1O,N,N)
      WRITE (6,162)
      CALL MWRITE (PKKM1O,N,N)
C
C
   36 IF (ISIGV.NE.0) GO TO 37
      CALL VREAD (SIGV,M)
      WRITE (6,163)
      CALL VWRITE (SIGV,M)
C
C
   37 IF (ISIGW.NE.0) GO TO 38
      CALL VREAD (SIGW,IN)
      WRITE (6,164)
      CALL VWRITE (SIGW,IN)
C
C
   38 IF (IXHZ.NE.0) GO TO 39
      CALL VREAD (XHATZ,N)
      WRITE (6,165)
      CALL VWRITE (XHATZ,N)
C
C
   39 IF (IC.EQ.1) GO TO 40
C     IC.NE.1 MBANS THAT MEANS AND STD. DEVIATIONS OF THE INITIAL STATE
C     VALLE MUST BE READ IN. OTHERWISE NOT REAC IN.
      CALL VREAD (XZMEAN,N)
      WRITE (6,166)
      CALL VWRITE (XZMEAN,N)
C
C
      CALL VREAD (SIGXZ,N)
      WRITE (6,167)
      CALL VWRITE (SIGXZ,N)
      GC TO 45
C
C
```

```
   40 READ (5,169) (XS(I,1),I=1,9)                                      MCSP 337
C     INITIAL CONDITION HAS BEEN READ                                   MCSP 338
      IF (ITRK.NE.1) GO TO 45                                           MCSP 339
      IF (ITRC.NE.0) GO TO 43                                           MCSP 340
C                                                                       MCSP 341
      DO 41 K=2,NSAM                                                    MCSP 342
   41 READ (5,169) (XS(I,K),I=1,9)                                      MCSP 343
C                                                                       MCSP 344
      IF (N.GT.6) GO TO 44                                              MCSP 345
C                                                                       MCSP 346
C                                                                       MCSP 347
      DO 42 K=1,NSAM                                                    MCSP 348
      XS(3,K) = XS(4,K)                                                 MCSP 349
      XS(4,K) = XS(5,K)                                                 MCSP 350
      XS(5,K) = XS(7,K)                                                 MCSP 351
   42 XS(6,K) = XS(8,K)                                                 MCSP 352
C                                                                       MCSP 353
C                                                                       MCSP 354
C                                                                       MCSP 355
      WRITE (6,168)                                                     MCSP 356
      WRITE (6,171) (XS(I,1),I=1,N)                                     MCSP 357
      GO TO 44                                                          MCSP 358
   43 CALL TRACK                                                        MCSP 359
C     IF TRACK CALLED HERE IT SHOULD BE WRITTEN TO GENERATE AND         MCSP 360
C     STORE THE TRACK IN XS(N,K) FOR K=2(NOTE),NSAM                     MCSP 361
C                                                                       MCSP 362
   44 WRITE (6,170)                                                     MCSP 363
      WRITE (6,171) (XS(I,1),I=1,N)                                     MCSP 364
      WRITE (6,171) (XS(I,NSAM),I=1,N)                                  MCSP 365
   45 CONTINUE                                                          MCSP 366
C                                                                       MCSP 367
C                                                                       MCSP 368
C     THE FOLLOWING SECTION PREPARES FOR THE MONTE CARLO LOOP           MCSP 369
C                                                                       MCSP 370
C     FORM NXN IDENTITY MATRIX IN DOUBLE PRECISION                      MCSP 371
C                                                                       MCSP 372
C                                                                       MCSP 373
      DO 46 I=1,N                                                       MCSP 374
C                                                                       MCSP 375
      DO 46 J=1,N                                                       MCSP 376
      EI(I,J) = 0.D0                                                    MCSP 377
   46 IF (I.EQ.J) EI(I,J)=1.D0                                          MCSP 378
C                                                                       MCSP 379
C                                                                       MCSP 380
C     GIVEN THE MATRIX GAMMA AND THE COVARIANCE OF W COMPUTE Q          MCSP 381
C     USING DOUBLE PRECISION ARITHMETIC                                 MCSP 382
      IF (IQ.NE.1) GO TO 47                                             MCSP 383
      CALL GMAT                                                         MCSP 384
```

```
      WRITE (6,160)
      CALL MWRITE (Q,N,N)
   47 IF (IG.NE.-1) GO TO 49
C
C
      DC 48 K=1,NSAM
C
      DC 48 I=1,N
   48 READ (5,169) (GKS(I,J,K),J=1,M)
C
      GC TO 52
   49 IF (IG.NE.0) GO TO 52
C
C
      DO 51 K=1,NSAM
      CALL GAIN
C
C
      DC 51 I=1,N
C
      DC 50 L=1,N
   5C PKKS(I,L,K) = PKK(I,L)
C
C
      DC 51 J=1,M
   51 GKS(I,J,K) = G(I,J)
C
   52 CCNTINUE
C     IF GAINS WERE TO BE READ IN (IG=-1) OR COMPUTED ONLY
C     ONCE(IG=0), THIS HAS NOW BEEN DCNE
C
C
C     SET UP ARRAYS FOR COMPUTING STATISTICS
C
C
      DC 54 I=1,NENS
C
      DC 53 J=1,NSAM
   53 MISSD(I,J) = 0.0D0
C
   54 CCNTINUE
C
C
      DC 55 K=1,NSAM
      YP(K) = 0.0
      XP(K) = 0.0
      XINT(K) = 0.0
      TEMPA(K) = 0.0
```

```
C                                                                       MCSP 433
      DO 55 J=1,N                                                       MCSP 434
      XM(J,K) = 0.                                                      MCSP 435
      ERR(J,K) = 0.                                                     MCSP 436
C                                                                       MCSP 437
      DO 55 L=1,N                                                       MCSP 438
   55 VAR(J,L,K) = 0.                                                   MCSP 439
C                                                                       MCSP 440
C                                                                       MCSP 441
C                                                                       MCSP 442
C     BEGIN MAIN ITERATION LOOP HERE                                    MCSP 443
C                                                                       MCSP 444
C                                                                       MCSP 445
C                                                                       MCSP 446
      DO 62 ITER=1,NENS                                                 MCSP 447
      IF (IC.EQ.1) GO TO 56                                             MCSP 448
      CALL XZERO                                                        MCSP 449
C                                                                       MCSP 450
   56 DO 57 I=1,N                                                       MCSP 451
   57 XHKKM1(I) = XHATZ(I)                                              MCSP 452
C                                                                       MCSP 453
C                                                                       MCSP 454
      DO 61 K=1,NSAM                                                    MCSP 455
C                                                                       MCSP 456
C     FORM NOISY MEASUREMENT FROM TRUE STATE VALUE                      MCSP 457
C                                                                       MCSP 458
C                                                                       MCSP 459
      DO 58 I=1,N                                                       MCSP 460
   58 X(I) = XS(I,K)                                                    MCSP 461
C                                                                       MCSP 462
C                                                                       MCSP 463
      CALL MEAS                                                         MCSP 464
C     GAIN IS NOT TO BE COMPUTED ON-LINE IF IG.NE.1                     MCSP 465
      IF (IG.NE.1) GO TO 60                                             MCSP 466
C                                                                       MCSP 467
C     Q IS TO BE COMPUTED ON-LINE IF IFLQ.NE.0                          MCSP 468
      IF (IFLQ.NE.0) CALL QON                                           MCSP 469
C     R IS TO BE COMPUTED ON-LINE IF IFLR.NE.0                          MCSP 470
C                                                                       MCSP 471
      IF (IFLR.NE.0) CALL RON                                           MCSP 472
      CALL GAIN                                                         MCSP 473
C                                                                       MCSP 474
C                                                                       MCSP 475
      DO 59 I=1,N                                                       MCSP 476
C                                                                       MCSP 477
      DO 59 J=1,M                                                       MCSP 478
   59 GKS(I,J,K) = G(I,J)                                               MCSP 479
C                                                                       MCSP 480
```

```
C
C     UPDATE THE STATE ESTIMATE
C
   60 CALL ESTIM
C
C     UPDATE RUNNING SUMS USED IN COMPUTING STATISTICS
      CALL STAT
      IF (IMISS.NE.1) CALL KILL
      IF (K.EQ.NSAM) GO TO 62
C
C     UPDATE TRACK BY COMPUTING X(K+1)
      IF (ITRK.NE.1) CALL TRACK
   61 CONTINUE
C
C
   62 CONTINUE
C
C
C     DIVIDE RUNNING SUMS COMPUTED BY SUBROUTINE STAT BY ENSEMBLE
C     SIZE TO COMPUTE STATISTICS
C
C
      ENS = NENS
C
      DO 64 K=1,NSAM
C
      DO 64 J=1,N
      IF (ITRK.EQ.1) GO TO 63
      XM(J,K) = XM(J,K)/ENS
   63 ERR(J,K) = ERR(J,K)/ENS
   64 VAR(J,J,K) = VAR(J,J,K)/ENS-ERR(J,K)**2
C
C
C
      IF (ISTAT.EQ.0) GO TO 66
C
C     COMPUTE OFF-DIAGONAL TERMS IN COVARIANCE OF ESTIMATION
C     ERROR MATRIX IF ISTAT.NE.0
C
C
      DO 65 K=1,NSAM
C
      DO 65 L=2,N
      LM1 = L-1
C
      DO 65 J=1,LM1
   65 VAR(L,J,K) = VAR(L,J,K)/ENS-ERR(L,K)*ERR(J,K)
C
```

```
   66 CONTINUE                                                          MCSP 529
C                                                                       MCSP 530
C                                                                       MCSP 531
      IF (IPRT.NE.0) GO TO 72                                           MCSP 532
      WRITE (6,172)                                                     MCSP 533
C     WRITE GAINS, THEORETICAL COVARIANCES OF ESTIMATION ERROR          MCSP 534
C     IF ONE SET OF GAINS HAS BEEN USED.                                MCSP 535
      IF (IG.NE.0) GO TO 69                                             MCSP 536
      WRITE (6,173)                                                     MCSP 537
C                                                                       MCSP 538
      DO 67 K=1,NSAM                                                    MCSP 539
      WRITE (6,174) K                                                   MCSP 540
C                                                                       MCSP 541
      DO 67 I=1,N                                                       MCSP 542
   67 WRITE (6,171) (GKS(I,J,K),J=1,M)                                  MCSP 543
C                                                                       MCSP 544
      WRITE (6,175)                                                     MCSP 545
C                                                                       MCSP 546
      DO 68 K=1,NSAM                                                    MCSP 547
      WRITE (6,176) K                                                   MCSP 548
C                                                                       MCSP 549
      DO 68 I=1,N                                                       MCSP 550
   68 WRITE (6,171) (PKKS(I,J,K),J=1,N)                                 MCSP 551
C                                                                       MCSP 552
   69 WRITE (6,181)                                                     MCSP 553
      WRITE (6,177)                                                     MCSP 554
      WRITE (6,178)                                                     MCSP 555
C                                                                       MCSP 556
      DO 70 K=1,NSAM                                                    MCSP 557
      WRITE (6,180)                                                     MCSP 558
C                                                                       MCSP 559
      DO 70 I=1,N                                                       MCSP 560
   70 WRITE (6,179) K,I,XM(I,K),ERR(I,K),VAR(I,I,K)                     MCSP 561
C                                                                       MCSP 562
      WRITE (6,181)                                                     MCSP 563
      IF (ISTAT.EQ.0) GO TO 72                                          MCSP 564
      WRITE (6,182)                                                     MCSP 565
C                                                                       MCSP 566
      DO 71 K=1,NSAM                                                    MCSP 567
      WRITE (6,183) K                                                   MCSP 568
C                                                                       MCSP 569
      DO 71 I=1,N                                                       MCSP 570
   71 WRITE (6,171) (VAR(I,L,K),L=1,I)                                  MCSP 571
C                                                                       MCSP 572
      WRITE (6,181)                                                     MCSP 573
   72 IF (IPLT.NE.0) GO TO 88                                           MCSP 574
C                                                                       MCSP 575
      DO 73 K=1,NSAM                                                    MCSP 576
```

```
   73 XP(K) = K                                                         MCSP 577
C                                                                       MCSP 578
      IF (IGPLT.NE.1) GO TO 76                                          MCSP 579
C                                                                       MCSP 580
      DO 75 I=1,N                                                       MCSP 581
C                                                                       MCSP 582
      DO 75 J=1,M                                                       MCSP 583
C                                                                       MCSP 584
      DO 74 K=1,NSAM                                                    MCSP 585
   74 YP(K) = GKS(I,J,K)                                                MCSP 586
C                                                                       MCSP 587
      WRITE (6,181)                                                     MCSP 588
      CALL  PLOTP (XP,YP,NSAM,0)                                        MCSP 589
   75 WRITE (6,184) I,J                                                 MCSP 590
C                                                                       MCSP 591
   76 IF (ITHVPL.NE.1) GO TO 79                                         MCSP 592
C                                                                       MCSP 593
      DO 78 I=1,N                                                       MCSP 594
C                                                                       MCSP 595
      DO 77 K=1,NSAM                                                    MCSP 596
   77 YP(K) = PKKS(I,I,K)                                               MCSP 597
C                                                                       MCSP 598
      WRITE (6,181)                                                     MCSP 599
      CALL  PLOTP (XP,YP,NSAM,0)                                        MCSP 600
   78 WRITE (6,185) I,I                                                 MCSP 601
C                                                                       MCSP 602
   79 IF (IMTPLT.NE.1) GO TO 82                                         MCSP 603
C                                                                       MCSP 604
      DO 81 I=1,N                                                       MCSP 605
C                                                                       MCSP 606
      DO 80 K=1,NSAM                                                    MCSP 607
   80 YP(K) = XM(I,K)                                                   MCSP 608
C                                                                       MCSP 609
      WRITE (6,181)                                                     MCSP 610
      CALL  PLOTP (XP,YP,NSAM,0)                                        MCSP 611
   81 WRITE (6,186) I                                                   MCSP 612
C                                                                       MCSP 613
   82 IF (ISMPLT.NE.1) GO TO 85                                         MCSP 614
C                                                                       MCSP 615
      DO 84 I=1,N                                                       MCSP 616
C                                                                       MCSP 617
      DO 83 K=1,NSAM                                                    MCSP 618
   83 YP(K) = ERR(I,K)                                                  MCSP 619
C                                                                       MCSP 620
      WRITE (6,181)                                                     MCSP 621
      CALL  PLOTP (XP,YP,NSAM,0)                                        MCSP 622
   84 WRITE (6,187) I,I                                                 MCSP 623
C                                                                       MCSP 624
```

```
   85 IF (ISVPLT.NE.1) GO TO 88                                       MCSP 625
C                                                                     MCSP 626
      DC 87 I=1,N                                                     MCSP 627
C                                                                     MCSP 628
      DC 86 K=1,NSAM                                                  MCSP 629
   86 YF(K) = VAR(I,I,K)                                              MCSP 630
C                                                                     MCSP 631
      WRITE (6,181)                                                   MCSP 632
      CALL FLCTP (XP,YP,NSAM,0)                                       MCSP 633
   87 WRITE (6,188) I                                                 MCSP 634
C                                                                     MCSP 635
      WRITE (6,181)                                                   MCSP 636
C                                                                     MCSP 637
C     THE FOLLOWING SECTION PLOTS THE DESIRED MISS DISTANCE INFORMATION.MCSP 638
C                                                                     MCSP 639
   88 IF (IMISS.NE.0) GO TO 105                                       MCSP 640
C                                                                     MCSP 641
      DC 90 K=1,NSAM                                                  MCSP 642
C                                                                     MCSP 643
      DO 89 J=1,NENS                                                  MCSP 644
   89 SMISS(J,K) = MISSD(J,K)                                         MCSP 645
C                                                                     MCSP 646
   90 CCNTINUE                                                        MCSP 647
C                                                                     MCSP 648
C                                                                     MCSP 649
      DC 92 K=1,NSAM                                                  MCSP 650
C                                                                     MCSP 651
      DC 91 J=1,NENS                                                  MCSP 652
   91 YP(K) = YP(K)+SMISS(J,K)                                        MCSP 653
C                                                                     MCSP 654
   92 CCNTINUE                                                        MCSP 655
C                                                                     MCSP 656
C                                                                     MCSP 657
      DC 93 K=1,NSAM                                                  MCSP 658
   93 YP(K) = YP(K)/ENS                                               MCSP 659
C                                                                     MCSP 660
      ENS1 = ENS-1.                                                   MCSP 661
C                                                                     MCSP 662
      DC 95 K=1,NSAM                                                  MCSP 663
C                                                                     MCSP 664
C                                                                     MCSP 665
      DC 94 I=1,NENS                                                  MCSP 666
   94 TEMPA(K) = TEMPA(K)+(SMISS(I,K)-YP(K))**2                       MCSP 667
C                                                                     MCSP 668
   95 TEMPA(K) = SQRT(TEMPA(K)/ENS1)                                  MCSP 669
C                                                                     MCSP 670
      IF (IMISSK.NE.0) GO TO 97                                       MCSP 671
C                                                                     MCSP 672
```

```
      DO 96 K=1,NSAM
   96 XP(K) = K
C
      WRITE (6,181)
      CALL PLOTP (XP,YP,NSAM,0)
      WRITE (6,199)
      WRITE (6,181)
      CALL PLOTP (XP,TEMPA,NSAM,0)
      WRITE (6,200)
   97 IF (IMISSR.NE.0) GO TO 99
C
      DO 98 K=1,NSAM
   98 XP(K) = XS(1,K)
C
      WRITE (6,181)
      CALL PLOTP (XP,YP,NSAM,0)
      WRITE (6,201)
      WRITE (6,181)
      CALL PLOTP (XP,TEMPA,NSAM,0)
      WRITE (6,202)
   99 IF (IMISSI.NE.0) GO TO 101
C
      DO 100 K=1,NSAM
  100 XINT(K) = XINT(K)/ENS
C
      WRITE (6,181)
      CALL PLOTP (XINT,YP,NSAM,0)
      WRITE (6,203)
      WRITE (6,181)
      CALL PLOTP (XINT,TEMPA,NSAM,0)
      WRITE (6,204)
      WRITE (6,181)
C
C     THE FOLLOWING SECTION PRINTS THE DESIRED FIRING AND MISS DISTANCE
C     DATA.
C
  101 IF (IFIRE.NE.0) GO TO 102
      IF (NFIRE.EQ.0) GO TO 104
      RDS = NFIRE/ENS
      IRDS = RDS+0.5
      HIT = NHIT/ENS
      IHIT = HIT+0.5
      SCARE = NSCARE/ENS
      ISCARE = SCARE+0.5
      AMISS = NMISS/ENS
      IMISSN = AMISS+0.5
      PERHIT = (HIT/RDS)*100.
      PERSCR = (SCARE/RDS)*100.
```

```
      PERMIS = (AMISS/RDS)*100.                                         MCSP 721
      WRITE (6,195) IRDS                                                MCSP 722
      WRITE (6,196) IHIT,PERHIT                                         MCSP 723
      WRITE (6,197) ISCARE,PERSCR                                       MCSP 724
      WRITE (6,198) IMISSN,PERMIS                                       MCSP 725
  102 IF (IMISSC.NE.0) GO TO 105                                        MCSP 726
      WRITE (6,181)                                                     MCSP 727
      WRITE (6,205)                                                     MCSP 728
C                                                                       MCSP 729
      DO 103 K=1,NSAM                                                   MCSP 730
  103 WRITE (6,206) K,YP(K),TEMPA(K)                                    MCSP 731
C                                                                       MCSP 732
      GO TO 105                                                         MCSP 733
  104 WRITE (6,207)                                                     MCSP 734
  105 CONTINUE                                                          MCSP 735
      STOP                                                              MCSP 736
C                                                                       MCSP 737
C                                                                       MCSP 738
C                                                                       MCSP 739
  106 FORMAT (5(I10))                                                   MCSP 740
  107 FORMAT (I2)                                                       MCSP 741
  108 FORMAT (7(I10))                                                   MCSP 742
  109 FORMAT (2(I5))                                                    MCSP 743
  110 FORMAT (5(I10))                                                   MCSP 744
  111 FORMAT (15I5)                                                     MCSP 745
  112 FORMAT (20X,'DESCRIPTION OF RUN',//)                              MCSP 746
  113 FORMAT (10X,'GAINS COMPUTED OFF-LINE AND READ IN',/)              MCSP 747
  114 FORMAT (10X,'GAINS COMPUTED ONCE IN "GAIN" BEFORE STARTING MONTE CMCSP 748
     1ARLO',/)                                                          MCSP 749
  115 FORMAT (10X,'GAINS COMPUTED FOR EACH MEMBER OF ENSEMBLE',/)       MCSP 750
  116 FORMAT (10X,'THE STANDARD LINEAR EQS. DO NOT CHARACTERIZE THE FILTMCSP 751
     1ER',/)                                                            MCSP 752
  117 FORMAT (10X,'THE STD. KALMAN EQS. CHARACTERIZE THE LINEAR FILTER',MCSP 753
     1/)                                                                MCSP 754
  118 FORMAT (10X,'ONLY ONE TRACK IS USED AND IT IS GENERATED BY SUBROUTMCSP 755
     1INE TRACK',/)                                                     MCSP 756
  119 FORMAT (10X,'ONLY ONE TRACK IS USED AND IT IS READ IN',/)         MCSP 757
  120 FORMAT (10X,'SEVERAL TRACKS USED BUT NOT GENERATED FROM STD. LINEAMCSP 758
     1R DIFFERENCE EQS.',/)                                             MCSP 759
  121 FORMAT (10X,'SEVERAL TRACKS GENERATED BY USING THE STD. LINEAR DIFMCSP 760
     1FERENCE EQS.',/)                                                  MCSP 761
  122 FORMAT (10X,'MEAN OF TRACK, MEAN OF EST. ERROR AND COVARIANCE OF EMCSP 762
     1ST. ERROR ARE COMPUTED',/)                                        MCSP 763
  123 FORMAT (10X,'MEAN OF TRACK, MEAN AND VARRIANCES OF EST. ERROR ARE MCSP 764
     1COMPUTED',/)                                                      MCSP 765
  124 FORMAT (10X,'THE Q MATRIX IS COMPUTED ON-LINE AT EACH SAMPLE BY "QMCSP 766
     1ON"',/)                                                           MCSP 767
  125 FORMAT (10X,'THE COVARIANCE OF W IS READ IN AND Q IS COMPUTED BY "MCSP 768
```

```
   1CMAT" BEFORE STARTING MONTE CARLC',/)                            MCSP 769
126 FCRMAT (10X,'THE Q MATRIX IS READ IN',/)                          MCSP 770
127 FORMAT (10X,'R IS COMPUTED ON-LINE AT EACH SAMPLE BY "RON"',/)    MCSP 771
128 FORMAT (10X,'R IS READ IN',/)                                     MCSP 772
129 FCRMAT (////,20X,'INPUT DATA CALLED FOR',///)                     MCSP 773
13C FCRMAT (10X,'PHI MATRIX',/)                                       MCSP 774
131 FCRMAT (10X,'H MATRIX',/)                                         MCSP 775
132 FCRMAT (10X,'R MATRIX',/)                                         MCSP 776
133 FORMAT (10X,'COVARIANCE OF W',/)                                  MCSP 777
134 FORMAT (10X,'Q MATRIX',/)                                         MCSP 778
135 FORMAT (10X,'GAMMA MATRIX',/)                                     MCSP 779
136 FCRMAT (10X,'STANDARD DEVIATIONS CF MEASUREMENT NOISE',/)         MCSP 780
137 FORMAT (10X,'STANDARD DEVIATIONS CF INPUT FCRCING W',/)           MCSP 781
13E FCRMAT (10X,'XHAT(0/-1)',/)                                       MCSP 782
139 FCRMAT (10X,'MEANS AND VARIANCES OF X(0)',/)                      MCSP 783
140 FCRMAT (10X,'P(0/-1)',/)                                          MCSP 784
141 FCRMAT (////,20X,'OUTPUT CALLED FOR',///)                         MCSP 785
142 FORMAT (10X,'PRINTED OUTPUT OF THE FOLLOWING DATA',/)             MCSP 786
143 FCRMAT (15X,'GAIN MATRICES AND THEORETICAL COVARIANCE OF EST. ERROMCSP 787
   1R MATRICES',/)                                                    MCSP 788
144 FORMAT (15X,'SAMPLE MEANS OF TRACK AND ESTIMATION ERROR,SAMPLE VARMCSP 789
   1IANCES CF EST. ERRCR',/)                                          MCSP 790
145 FCRMAT (15X,'COVARIANCE OF ESTIMATICN ERROR MATRIX',/)            MCSP 791
146 FCRMAT (10X,'NO PRINTED MONTE CARLO OUTPUT IS DESIRED',///)       MCSP 792
147 FORMAT (10X,'THE FOLLOWING PLOTS ARE CALLED FOR',/)               MCSP 793
148 FCRMAT (15X,'G(K) VS. K',/)                                       MCSP 794
149 FORMAT (15X,'P(K/K) THEORETICAL VS. K',/)                         MCSP 795
150 FCRMAT (15X,'MEAN OF TRACK VS. K',/)                              MCSP 796
151 FORMAT (15X,'SAMPLE MEANS OF ESTIMATION ERRCR VS. K',/)           MCSP 797
152 FORMAT (15X,'SAMPLE VARIANCES OF ESTIMATICN ERROR VS. K',/)       MCSP 798
153 FCRMAT (10X,'NO PLOTTED MONTE CARLO OUTPUT IS CALLED FOR',/)      MCSP 799
154 FORMAT (20X,'INPUT DATA',///)                                     MCSP 800
155 FORMAT (4X,'N=',I2,4X,'M=',I2,4X'IN=',I2,4X'NSAM=',I3,4X,'NENS=',IMCSP 801
   15,4X'ND=',I2,//)                                                  MCSP 802
156 FORMAT (//,10X,'THE PHI MATRIX IS',/)                             MCSP 803
157 FCRMAT (//,10X,'THE H MATRIX IS',/)                               MCSP 804
158 FCRMAT (//,10X,'THE R MATRIX IS',/)                               MCSP 805
159 FORMAT (//,10X,'THE COVARIANCE CF W MATRIX IS',/)                 MCSP 806
160 FCRMAT (//,10X,'THE Q MATRIX IS',/)                               MCSP 807
161 FCRMAT (//,10X,'THE GAMMA MATRIX IS',/)                           MCSP 808
162 FCRMAT (//,10X,'THE MATRIX P(0/-1) IS',/)                         MCSP 809
163 FCRMAT (//,10X,'THE STD. DEVIATICNS OF MEASUREMENT NOISE ARE',/)  MCSP 810
164 FORMAT (//,10X,'THE STD. DEVIATIONS OF INPUT FORCING W ARE',/)    MCSP 811
165 FCRMAT (//,10X,'THE VECTOR XHAT(0/-1) IS',/)                      MCSP 812
166 FCRMAT (//,10X,'THE MEAN OF THE VECTOR X(0) IS',/)                MCSP 813
167 FCRMAT (//,10X,'THE STANDARD DEVIATIONS CF THE VECTOR X(0) ARE',/)MCSP 814
168 FCRMAT (//,10X,'THE INITIAL STATE IS',/)                          MCSP 815
169 FCRMAT (4(1PE20.12))                                              MCSP 816
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170 FCRMAT (//,10X,'THE FIRST AND LAST POINTS ON THE SINGLE TRACK TO BMCSP 817
    1E USED ARE',/)                                                   MCSP 818
171 FCRMAT (9(2X,1PE12.5),/)                                          MCSP 819
172 FCFMAT ('1',20X,'OUTPUT DATA',///)                                MCSP 820
173 FORMAT (10X,'THE GAIN MATRICES ARE',/)                            MCSP 821
174 FCRMAT (5X,'K=',I3,/,10X,'G(K)=',/)                               MCSP 822
175 FORMAT (1X,//,10X,'THE THEORETICAL COVARIANCE MATRIX IS',/)       MCSP 823
176 FCRMAT (5X,'K=',I3,/,10X,'P(K/K)=',/)                             MCSP 824
177 FCRMAT (T5,'TIME',T16,'VECTCR COM-',T34,'SAMPLE MEAN',            MCSP 825
    1 T51,'SAMPLE MEAN OF',T71,'SAMPLE VARIANCE CF')                  MCSP 826
178 FCRMAT (T5,'INDEX',T16,'PONENT INDEX',T34,'CF TRACK',             MCSP 827
    1 T51,'ESTIMATION ERROR',T71,'ESTIMATION ERRCR')                  MCSP 828
179 FORMAT (6X,I3,13X,I1,10X,1PE14.7,2(6X,1PE14.7))                   MCSP 829
180 FCRMAT (//)                                                       MCSP 830
181 FCRMAT ('1')                                                      MCSP 831
182 FCRMAT (10X,'THE SAMPLE COVARIANCE OF EST. ERROR MATRIX IS',//)   MCSP 832
183 FCRMAT (//,2X,'K=',I3,/)                                          MCSP 833
184 FCRMAT (12X,'G(',I1,',',I1,')  VS. K')                            MCSP 834
185 FCRMAT (12X,'PKK(',I1,',',I1,')  VS. K')                          MCSP 835
186 FCRMAT (12X,'MEAN OF X(',I1,')  VS. K')                           MCSP 836
187 FCRMAT (12X,'XHATKK(',I1,') -X(',I1,')  VS. K')                   MCSP 837
188 FCRMAT (12X,'ERROR VARIANCE(',I1,') VS.. K')                      MCSP 838
189 FGRMAT (15X,'MEAN AND STANDARD CEVIATION OF MISS DISTANCE VS. K',/MCSP 839
    1)                                                                MCSP 840
190 FORMAT (15X,'MEAN AND STANDARD DEVIATION OF MISS DISTANCE VS. SLANMCSP 841
    1T RANGE',/)                                                      MCSP 842
191 FCRMAT (15X,'MEAN AND STANDARD CEVIATION OF MISS DISTANCE VS AVERAMCSP 843
    1GE LENGTH OF PREDICTION INTERVAL',/)                             MCSP 844
192 FCRMAT (15X,'AVERAGE NUMBER CF RCUNDS FIRED, HITS,SCARES,AND MISSEMCSP 845
    1S PER ENSEMBLE RUN',/)                                           MCSP 846
193 FCRMAT (15X,'MEAN AND STANDARD DEVIATION OF THE MISS DISTANCE FOR MCSP 847
    1EACH K',/)                                                       MCSP 848
194 FCRMAT (10X,'NO FIRING OR MISS DISTANCE DATA IS DESIRED',/)       MCSP 849
195 FORMAT (15X,'AVERAGE NUMBER OF RCUNDS FIRED PER ENSEMBLE RUN =',I3MCSP 850
    1,/)                                                              MCSP 851
196 FCRMAT (15X,'AVERAGE NUMBER OF HITS PER ENSEMBLE RUN =',I3,2X,'OR'MCSP 852
    1,2X,F6.2,1X,'PERCENT',/)                                         MCSP 853
197 FORMAT (15X,'AVERAGE NUMBER OF SCARES PER ENSEMBLE RUN =',I3,2X,'CMCSP 854
    1R',F6.2,1X,'PERCENT',/)                                          MCSP 855
198 FCRMAT (15X,'AVERAGE NUMBER CF MAGGIES DRAWERS PER ENSEMBLE RUN ='MCSP 856
    1,I3,2X,'OR',F6.2,1X,'PERCENT',/)                                 MCSP 857
199 FORMAT (15X,'MEAN OF MISS DISTANCE VS. K',/)                      MCSP 858
200 FORMAT (15X,'STANDARD DEVIATION OF MISS DISTANCE VS. K',/)        MCSP 859
201 FORMAT (15X,'MEAN OF MISS DISTANCE VS. SLANT RANGE',/)            MCSP 860
202 FORMAT (15X,'STANDARD DEVIATION OF MISS DISTANCE VS. SLANT RANGE',MCSP 861
    1/)                                                               MCSP 862
203 FCRMAT (15X,'MEAN OF MISS DISTANCE VS. AVERAGE LENGTH OF PREDICTIOMCSP 863
    1N INTERVAL',/)                                                   MCSP 864
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204 FORMAT (15X,'STANDARD DEVIATION OF MISS DISTANCE VS. AVERAGE LENGTMCSP 865
   1H OF PREDICTION INTERVAL',/)                                      MCSP 866
205 FORMAT (14X,'K',5X,'MEAN OF MISS DIST.',2X,'STD. DEV. OF MISS DISTMCSP 867
   1.',///)                                                           MCSP 868
206 FORMAT (12X,I3,7X,1PE13.6,10X,1PE13.6,/)                          MCSP 869
207 FORMAT (///,15X,'NO ROUNDS WERE FIRED; GO DIRECTLY TO THE SHIPYARDMCSP 870
   1, DO NOT COLLECT KILL',/)                                         MCSP 871
208 FORMAT (10X,'THE FOLLOWING MISS DISTANCE DATA IS PLOTTED',/)      MCSP 872
209 FORMAT (10X,'NO PLOTTED MISS DISTANCE DATA IS CALLED FOR',/)      MCSP 873
    END                                                               MCSP 874
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```
      SUBROUTINE ESTIM                                                  ESTI   1
C     THIS SUBROUTINE UPDATES THE STATE ESTIMATE. IN THE DEFAULT        ESTI   2
C     CCNDITION (IEST.EQ.0) THE STANDARD EQUATIONS                      ESTI   3
C                                                                       ESTI   4
C     XHAT(K/K)=XHAT(K/K-1)+G(K)*(Z(K)-H(5)*XHAT(K/K-1))                ESTI   5
C                                                                       ESTI   6
C     XHAT(K+1/K)=PHI*XHAT(K/K)                                         ESTI   7
C                                                                       ESTI   8
C     ARE EVALUATED                                                     ESTI   9
      REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI,PKKM10ESTI  10
     1,MISSD                                                            ESTI  11
      CCMMON GAMMA(9,9),COVW(9,9),R(9,9),PHI(9,9),H(9,9),TEMP(9,9),TEMP1ESTI  12
     1(9,9),TEMP2(9,9),PKKM1(9,9),PKKM10(9,9),G(9,9),PKK(9,9),Q(9,9),EI(ESTI  13
     29,9),MISSD(100,160),VAR(9,9,160),GKS(9,9,160),PKKS(9,9,160),XM(9,1ESTI  14
     360),ERR(9,160),GAMMAS(9,9),PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGESTI  15
     4W(9),X(9),SIGXZ(9),XZMEAN(9),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SESTI  16
     5IGV(9),XHATZ(9),XINT(160),N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXESTI  17
     6Z,IV,IW,IEST,ND,NHIT,NSCARE,NMISS,NFIRE                           ESTI  18
C     TAKE THE APPROPRIATE GAIN AND STORE IN THE ARRAY GK               ESTI  19
C                                                                       ESTI  20
C                                                                       ESTI  21
      DC 1 I=1,N                                                        ESTI  22
C                                                                       ESTI  23
C                                                                       ESTI  24
      DC 1 J=1,M                                                        ESTI  25
    1 GK(I,J) = GKS(I,J,K)                                              ESTI  26
C                                                                       ESTI  27
C                                                                       ESTI  28
      IF (IEST.NE.0) GO TO 2                                            ESTI  29
      CALL VPROD (HS,XHKKM1,M,N,VTMP)                                   ESTI  30
      CALL VSUB (Z,VTMP,M,VTMP)                                         ESTI  31
      CALL VPROD (GK,VTMP,N,M,VTMP)                                     ESTI  32
      CALL VADD (XHKKM1,VTMP,N,XHKK)                                    ESTI  33
C                                                                       ESTI  35
C     XHAT(K/K) HAS BEEN COMPUTED AND STORED IN THE ARRAY XHKK          ESTI  36
      CALL VPROD (PHIS,XHKK,N,N,XHKKM1)                                 ESTI  37
C                                                                       ESTI  39
C     XHAT(K+1/K) HAS BEEN COMPUTED AND STORED IN THE ARRAY XHKKM1      ESTI  40
      RETURN                                                            ESTI  41
    2 CCNTINUE                                                          ESTI  42
C     IF STANDARD EQUATIONS ARE NCT TO BE USED, THE APPROPRIATE         ESTI  43
C     EQUATIONS MUST BE INSERTED HERE BY THE USER.                      ESTI  44
      RETURN                                                            ESTI  45
      END                                                               ESTI  46
```

```
      SUBROUTINE GAIN                                                   GAIN
      REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI,PKKM10GAIN
     1,MISSD                                                            GAIN
      COMMON GAMMA(9,9),COVW(9,9),R(9,9),PHI(9,9),H(9,9),TEMP(9,9),TEMP1GAIN
     1(9,9),TEMP2(9,9),PKKM1(9,9),PKKM10(9,9),G(9,9),PKK(9,9),Q(9,9),EI(GAIN
     29,9),MISSD(100,160),VAR(9,9,160),GKS(9,9,160),PKKS(9,9,160),XM(9,1GAIN
     360),ERR(9,160),GAMMAS(9,9),PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGGAIN
     4W(9),X(9),SIGXZ(9),XZMEAN(9),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SGAIN
     5IGV(9),XHATZ(9),XINT(160),N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXGAIN
     6Z,IV,IW,IEST,ND,NHIT,NSCARE,NMISS,NFIRE                           GAIN
C                                                                       GAIN
C     G(K) = P(K/K-1)*HT*(H*P(K/K-1)*HT + R)                            GAIN
      IF (K.NE.1) GO TO 2                                               GAIN
C                                                                       GAIN
C                                                                       GAIN
      DO 1 IL=1,N                                                       GAIN
C                                                                       GAIN
      DO 1 JL=1,N                                                       GAIN
    1 PKKM1(IL,JL) = PKKM10(IL,JL)                                      GAIN
C                                                                       GAIN
C                                                                       GAIN
    2 CONTINUE                                                          GAIN
      CALL TRANS (H,M,N,TEMP2)                                          GAIN
      CALL PROD (PKKM1,TEMP2,N,N,M,TEMP)                                GAIN
      CALL PROD (H,TEMP,M,N,M,TEMP1)                                    GAIN
      CALL ADD (TEMP1,R,M,M,TEMP1)                                      GAIN
      IF (M.EQ.1) GO TO 4                                               GAIN
      MD = ND                                                           GAIN
      CALL GAUSS3 (M,EPS,TEMP1,TEMP2,KER,MD)                            GAIN
      CALL PROD (TEMP,TEMP2,N,M,M,G)                                    GAIN
C                                                                       GAIN
C     NOTE HERE  PKK(I,J) = P(K/K)  WHERE                               GAIN
C     P(K/K) = (I-G(K)*H)*P(K/K-1)                                      GAIN
    3 CALL PROD (G,H,N,M,N,TEMP)                                        GAIN
      CALL SUB (EI,TEMP,N,N,TEMP2)                                      GAIN
      CALL PROD (TEMP2,PKKM1,N,N,N,PKK)                                 GAIN
C                                                                       GAIN
C     NOTE HERE  PKKM1(I,J) = P(K/K-1)  WHERE                           GAIN
C     P(K/K-1)= PHI*P(K-1/K-1)*PHIT + Q                                 GAIN
      CALL TRANS (PHI,N,N,TEMP2)                                        GAIN
      CALL PROD (PKK,TEMP2,N,N,N,TEMP)                                  GAIN
      CALL PROD (PHI,TEMP,N,N,N,TEMP1)                                  GAIN
      CALL ADD (TEMP1,Q,N,N,PKKM1)                                      GAIN
      RETURN                                                            GAIN
C                                                                       GAIN
C                                                                       GAIN
    4 DO 5 I=1,N                                                        GAIN
    5 G(I,1) = TEMP(I,1)/TEMP1(1,1)                                     GAIN
```

```
C                                                                       GAIN  49
C                                                                       GAIN  50
      GO TO 3                                                           GAIN  51
      END                                                               GAIN  52
```

```
      SUBROUTINE QMAT
C
C     THIS SUBROUTINE COMPUTES THE MATRIX Q FROM THE EQUATION
C
C        Q=GAMMA* E(W*WT) * GAMMAT
C
C     DOUBLE PRECISION ARITHMETIC IS USED
C
C
      REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI,PKKM10
     1,MISSD
      COMMON GAMMA(9,9),COVW(9,9),R(9,9),PHI(9,9),H(9,9),TEMP(9,9),TEMP1
     1(9,9),TEMP2(9,9),PKKM1(9,9),PKKM10(9,9),G(9,9),PKK(9,9),Q(9,9),EI(
     29,9),MISSD(100,160),VAR(9,9,160),GKS(9,9,160),PKKS(9,9,160),XM(9,1
     360),ERR(9,160),GAMMAS(9,9),PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIG
     4W(9),X(9),SIGXZ(9),XZMEAN(9),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),S
     5IGV(9),XHATZ(9),XINT(160),N,NSAM,IG,M,ITER,ITRK,IN,ISTAT,K,ITRO,IX
     6Z,IV,IW,IEST,ND,NHIT,NSCARE,NMISS,NFIRE
C
C
      CALL PROD (GAMMA,COVW,N,IN,IN,TEMP)
      CALL TRANS (GAMMA,N,IN,TEMP1)
      CALL PROD (TEMP,TEMP1,N,IN,N,G)
      RETURN
      END
```

```
      SUBROUTINE MEAS                                                   MEAS
      REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI,PKKM10MEAS
     1,MISSD                                                            MEAS
      COMMON GAMMA(9,9),COVW(9,9),R(9,9),PHI(9,9),H(9,9),TEMP(9,9),TEMP1MEAS
     1(9,9),TEMP2(9,9),PKKM1(9,9),PKKM10(9,9),G(9,9),PKK(9,9),Q(9,9),EI(MEAS
     29,9),MISSD(100,160),VAR(9,9,160),GKS(9,9,160),PKKS(9,9,160),XM(9,1MEAS
     360),ERR(9,160),GAMMAS(9,9),PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGMEAS
     4W(9),X(9),SIGXZ(9),XZMEAN(9),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SMEAS
     5IGV(9),XHATZ(9),XINT(160),N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXMEAS
     6Z,IV,IW,IEST,ND,NHIT,NSCARE,NMISS,NFIRE                           MEAS
      N1 = N/3                                                          MEAS
      NN = N-1                                                          MEAS
      J = 0                                                             MEAS
C                                                                       MEAS
      DO 1 I=1,NN,N1                                                    MEAS
      J = J+1                                                           MEAS
    1 Z(J) = XS(I,K)                                                    MEAS
C                                                                       MEAS
      CALL SNORM (IV,V,M)                                               MEAS
C                                                                       MEAS
C                                                                       MEAS
      DO 2 I=1,M                                                        MEAS
    2 V(I) = SIGV(I)*V(I)                                               MEAS
C                                                                       MEAS
C                                                                       MEAS
      CALL VADD (Z,V,M,Z)                                               MEAS
      RETURN                                                            MEAS
      END                                                               MEAS
```

```
      SUBROUTINE KILL                                                   KILL   1
      REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI,PKKM10KILL   2
     1,MISSD,TEMPP1,PREPOS,VDTEMP,ER1,DELTAT,DELTAP,DELTR,D,CHORD1,ANGLEKILL   3
     2A,CHORD2,ANGLEB,ELEG1,ERRNET,PI,DEGRAD,B,TEMPX,TEMPX1,SUM1,SUM2   KILL   4
      COMMON GAMMA(9,9),COVW(9,9),R(9,9),PHI(9,9),H(9,9),TEMP(9,9),TEMP1KILL   5
     1(9,9),TEMP2(9,9),PKKM1(9,9),PKKM10(9,9),G(9,9),PKK(9,9),Q(9,9),EI(KILL   6
     29,9),MISSD(100,160),VAR(9,9,160),GKS(9,9,160),PKKS(9,9,160),XM(9,1KILL   7
     360),ERR(9,160),GAMMAS(9,9),PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGKILL   8
     4W(9),X(9),SIGXZ(9),XZMEAN(9),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SKILL   9
     5IGV(9),XHATZ(9),XINT(160),N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXKILL  10
     6Z,IV,IW,IEST,ND,NHIT,NSCARE,NMISS,NFIRE                           KILL  11
      DIMENSION TEMPP1(9,9),TEMPX(9),PREPOS(9,160),VDTEMP(9),ER1(3),TEMP
     1X1(9)
      DATA PI,DEGRAD,SAMPR,RING1,RING2/3.1415929 2635,57.2957795131,0.25,
     18.,16./
      N1 = N/3                                                          KILL  17
      N2 = 2*N1                                                         KILL  18
      N3 = N1+1                                                         KILL  19
      NN = N-1                                                          KILL  20
C                                                                       KILL  21
C     STORE THE CURRENT ESTIMATE XHAT(K/K) AND THE EXACT POSITION X(K)  KILL  22
C     FOR FUTURE USE.  CONVERT ALL ANGLES TO RADIAN MEASURE.            KILL  23
C                                                                       KILL  24
C                                                                       KILL  25
      DO 1 I=1,N                                                        KILL  26
      VDTEMP(I) = 0.0D0                                                 KILL  27
      TEMPX1(I) = 0.0D0                                                 KILL  28
      TEMPX1(I) = XS(I,K)                                               KILL  29
    1 PREPOS(I,K) = XHKK(I)                                             KILL  30
C                                                                       KILL  31
C                                                                       KILL  32
      DO 2 I=N3,N                                                       KILL  33
      TEMPX1(I) = TEMPX1(I)/DEGRAD                                      KILL  34
    2 PREPOS(I,K) = PREPOS(I,K)/DEGRAD                                  KILL  35
C                                                                       KILL  36
C                                                                       KILL  37
C     THIS SECTION COMPUTES THE TIME OF FLIGHT (TOF) OF THE PROJECTILE, KILL  38
C     FROM DATA GIVEN IN THE 5"/54 CAL RANGE TABLE.                     KILL  39
      TOF = (.143E-02)*XHKK(1)
C     DURING THE PROJECTILE TOF, N MEASUREMENTS WILL BE TAKEN.          KILL  49
      NSAMP = TOF/SAMPR
      TNSAMP = FLOAT(NSAMP)
C                                                                       KILL  48
C                                                                       KILL  53
C     IF THE PROJECTILE TOF IS GREATER THAN 10.00 SECS., THE TARGET     KILL  54
C     WILL BE CONSIDERED TO BE OUT OF RANGE.  NO ROUNDS WILL BE FIRED.  KILL  55
C     IF THE TARGET IS NOT TAKEN UNDER FIRE, ASSIGN A LARGE VALUE OF    KILL  56
C     MISS DISTANCE AS A PENALTY.                                       KILL  57
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C                                                                       KILL  58
      IF(NSAMP.GE.K) GO TO 3                                            KILL  59
      IF(TOF.LE.10.0) GO TO 31                                          KILL  60
    3 ERRNET = 6.0D 02                                                  KILL  61
      GO TO 17                                                          KILL  62
   31 CONTINUE                                                          KILL  63
      XINT(K) = XINT(K)+TNSAMP                                          KILL  64
C                                                                       KILL  65
C                                                                       KILL  66
      DO 4 I=1,N                                                        KILL  67
    4 TEMPX(I) = PREPOS(I,K-NSAMP)                                      KILL  68
C                                                                       KILL  69
C                                                                       KILL  70
C     THIS SECTION RAISES PHI TO THE NSAMP POWER.                       KILL  71
C                                                                       KILL  72
C                                                                       KILL  73
      DO 5 I=1,N                                                        KILL  74
C                                                                       KILL  75
      DO 5 J=1,N                                                        KILL  76
    5 TEMPP1(I,J) = 0.0D0                                               KILL  77
C                                                                       KILL  78
C                                                                       KILL  79
      TEMPP1(1,1) = 1.0D0                                               KILL  80
      TEMPP1(1,2) = TNSAMP*SAMPR                                        KILL  81
      TEMPP1(2,2) = 1.0D0                                               KILL  82
      IF (N.NE.9) GO TO 8                                               KILL  83
      TEMPP1(3,3) = PHI(3,3)**NSAMP                                     KILL  84
      SUM1 = 0.00                                                       KILL  85
      SUM2 = 0.00                                                       KILL  86
      NSAMP1 = NSAMP-1                                                  KILL  87
C                                                                       KILL  88
C                                                                       KILL  89
      DO 6 I=1,NSAMP                                                    KILL  90
      J = I-1                                                           KILL  91
    6 SUM1 = SUM1+(PHI(3,3)**J)                                         KILL  92
C                                                                       KILL  93
C                                                                       KILL  94
C                                                                       KILL  95
      DO 7 I=1,NSAMP1                                                   KILL  96
      J = I-1                                                           KILL  97
    7 SUM2 = SUM2+(TNSAMP-I)*(PHI(3,3)**J)                              KILL  98
C                                                                       KILL  99
C                                                                       KILL 100
      TEMPP1(2,3) = PHI(2,3)*SUM1                                       KILL 101
      TEMPP1(1,3) = PHI(1,3)*SUM1+PHI(1,2)*PHI(2,3)*SUM2                KILL 102
C                                                                       KILL 103
C                                                                       KILL 104
    8 DO 10 NA=N1,N2,N1                                                 KILL 105
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```
C                                                                       KILL 106
      DO 9 I=1,N1                                                       KILL 107
      INA = I+NA                                                        KILL 108
C                                                                       KILL 109
      DC 9 J=1,N1                                                       KILL 110
      JNA = J+NA                                                        KILL 111
    9 TEMPP1(INA,JNA) = TEMPP1(I,J)                                     KILL 112
C                                                                       KILL 113
   1C CCNTINUE                                                          KILL 114
C                                                                       KILL 115
C                                                                       KILL 116
C     FCRM THE PREDICTED POSITION, XHAT(K/K-NSAMP) = (PHI**NSAMP)*      KILL 117
C     XHAT(K-NSAMP/K-NSAMP)                                             KILL 118
C                                                                       KILL 119
      DC 12 I=1,N                                                       KILL 120
C                                                                       KILL 121
      DC 11 J=1,N                                                       KILL 122
   11 VDTEMP(I) = VDTEMP(I)+TEMPP1(I,J)*TEMPX(J)                        KILL 123
C                                                                       KILL 124
   12 CCNTINUE                                                          KILL 125
C                                                                       KILL 126
C                                                                       KILL 127
C     CCMPUTE THE PREDICTION ERRORS AND THE MAGNITUDE OF THE ERROR      KILL 128
C     VECTOR.                                                           KILL 129
C                                                                       KILL 130
      J = 0                                                             KILL 131
C                                                                       KILL 132
      DO 13 I=1,NN,N1                                                   KILL 133
      J = J+1                                                           KILL 134
   13 ER1(J) = TEMPX1(I)-VDTEMP(I)                                      KILL 135
C                                                                       KILL 163
C                                                                       KILL 136
C                                                                       KILL 137
      DELTR = DABS(ER1(1))                                              KILL 138
      DELTAP = DABS(ER1(2))                                             KILL 139
      DELTAT = DABS(ER1(3))                                             KILL 140
      D = DMAX1(TEMPX1(1),VDTEMP(1))                                    KILL 141
      CHORD1 = 2.*DSIN(DELTAP/2.)*D                                     KILL 142
      ANGLEA = (PI-DELTAP)/2.                                           KILL 143
      ELEG1 = ((CHORD1**2)+(DELTR**2)-2.*DELTR*CHCRD1*DCOS(ANGLEA))     KILL 144
      ANGLEB = (PI-DELTAT)/2.                                           KILL 145
      CHCRD2 = 2.*DSIN(DELTAT/2.)*D                                     KILL 146
      B = DSQRT(ELEG1)                                                  KILL 147
      ERRNET = DSQRT(ELEG1+((CHORD2)**2)-2.*B*(CHCRD2)*DCOS(ANGLEB))    KILL 148
C                                                                       KILL 150
C     CCMPUTE RCUNDS FIRED, HITS, MISSES AND SCARES                     KILL 151
C                                                                       KILL 152
      IF (ERRNET.GT.RING2) GO TO 15                                     KILL 153
```

```
      IF (ERRNET.LE.RING1) GO TO 14                                   KILL 154
      NSCARE = NSCARE+1                                               KILL 155
      GC TO 16                                                        KILL 156
   14 NHIT = NHIT+1                                                   KILL 157
      GO TO 16                                                        KILL 158
   15 NMISS = NMISS+1                                                 KILL 159
   16 NFIRE = NFIRE+1                                                 KILL 160
   17 MISSD(ITER,K) = ERRNET                                          KILL 161
      RETURN                                                          KILL 162
      END                                                             KILL 166
```

```
      SUBROUTINE QON                                                    QONL
C                                                                       QONL
C     IF Q IS TO BE COMPUTED ON-LINE (IFLQ.NE.0) IT IS DONE             QONL
C     IN THIS SUBROUTINE                                                QONL
C                                                                       QONL
      REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI,PKKM10QONL
     1,MISSD                                                            QONL
      COMMON GAMMA(9,9),COVW(9,9),R(9,9),PHI(9,9),H(9,9),TEMP(9,9),TEMP1QONL
     1(9,9),TEMP2(9,9),PKKM1(9,9),PKKM10(9,9),G(9,9),PKK(9,9),Q(9,9),EI(QONL
     29,9),MISSD(100,160),VAR(9,9,160),GKS(9,9,160),PKKS(9,9,160),XM(9,1QONL
     360),ERR(9,160),GAMMAS(9,9),PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGQONL
     4W(9),X(9),SIGXZ(9),XZMEAN(9),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SQONL
     5IGV(9),XHATZ(9),XINT(160),N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXQONL
     6Z,IV,IW,IEST,ND,NHIT,NSCARE,NMISS,NFIRE                           QONL
C                                                                       QONL
C     THE APPROPRIATE STATEMENTS FOR COMPUTING Q ON-LINE MUST           QONL
C     BE INSERTED HERE BY THE USER                                      QONL
C                                                                       QONL
      RETURN                                                            QONL
      END                                                               QONL
```

```
      SUBROUTINE RON                                                    RONL   1
C                                                                       RONL   2
C     IF R IS TO BE COMPUTED ON-LINE (IFLR.NE.0) IT IS DONE             RONL   3
C     IN THIS SUBROUTINE                                                RONL   4
C                                                                       RONL   5
      REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI,PKKM10RONL   6
     1,MISSD                                                            RONL   7
      COMMON GAMMA(9,9),COVW(9,9),R(9,9),PHI(9,9),H(9,9),TEMP(9,9),TEMP1RONL   8
     1(9,9),TEMP2(9,9),PKKM1(9,9),PKKM10(9,9),G(9,9),PKK(9,9),Q(9,9),EI(RONL   9
     29,9),MISSD(100,160),VAR(9,9,160),GKS(9,9,160),PKKS(9,9,160),XM(9,1RONL  10
     360),ERR(9,160),GAMMAS(9,9),PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGRONL  11
     4W(9),X(9),SIGXZ(9),XZMEAN(9),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SRONL  12
     5IGV(9),XHATZ(9),XINT(160),N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRC,IXRONL  13
     6Z,IV,IW,IEST,ND,NHIT,NSCARE,NMISS,NFIRE                           RONL  14
C                                                                       RONL  15
C     THE APPROPRIATE STATEMENTS FOR COMPUTING R ON-LINE MUST           RONL  16
C     BE INSERTED HERE BY THE USER                                      RONL  17
C                                                                       RONL  18
      RETURN                                                            RONL  19
      END                                                               RONL  20
```

```
      SUBROUTINE STAT                                                   STAT   1
C     THIS SUBROUTINE  COMPUTES RUNNING SUMS USED IN DETERMINING THE    STAT   2
C     SAMPLE STATISTICS OF TRACK AND ESTIMATION ERRORS. IN THE DEFAULT  STAT   3
C     OPTION (ISTAT.EQ.0) THE STATISTICS TO BE COMPUTED ARE MEAN OF     STAT   4
C     TRACK, MEAN OF ESTIMATION ERROR AND VARIANCE OF ESTIMATION        STAT   5
C     ERROR. IF(ISTAT.NE.0) THE OFF-DIAGONAL TERMS IN THE COVARIANCE OF STAT   6
C     ESTIMATION ERROR MATRIX ARE ALSO COMPUTED.                        STAT   7
      REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI,PKKM10STAT   8
     1,MISSD                                                            STAT   9
      COMMON GAMMA(9,9),COVW(9,9),R(9,9),PHI(9,9),H(9,9),TEMP(9,9),TEMP1STAT  10
     1(9,9),TEMP2(9,9),PKKM1(9,9),PKKM10(9,9),G(9,9),PKK(9,9),Q(9,9),EI(STAT  11
     29,9),MISSD(100,160),VAR(9,9,160),GKS(9,9,160),PKKS(9,9,160),XM(9,1STAT  12
     360),ERR(9,160),GAMMAS(9,9),PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGSTAT  13
     4W(9),X(9),SIGXZ(9),XZMEAN(9),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SSTAT  14
     5IGV(9),XHATZ(9),XINT(160),N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXSTAT  15
     6Z,IV,IW,IEST,ND,NHIT,NSCARE,NMISS,NFIRE                           STAT  16
      DIMENSION EXH(9)                                                  STAT  17
      IF (ITRK.NE.1) GO TO 2                                            STAT  18
      IF (ITER.NE.1) GO TO 4                                            STAT  19
C                                                                       STAT  20
C                                                                       STAT  21
      DO 1 J=1,N                                                        STAT  22
    1 XM(J,K) = XS(J,K)                                                 STAT  23
C                                                                       STAT  24
C                                                                       STAT  25
      GO TO 4                                                           STAT  26
    2 CONTINUE                                                          STAT  27
C                                                                       STAT  28
C                                                                       STAT  29
      DO 3 J=1,N                                                        STAT  30
    3 XM(J,K) = XM(J,K)+XS(J,K)                                         STAT  31
C                                                                       STAT  32
C                                                                       STAT  33
    4 CONTINUE                                                          STAT  34
C                                                                       STAT  35
C                                                                       STAT  36
      DO 5 J=1,N                                                        STAT  37
      EXH(J) = XHKK(J)-XS(J,K)                                          STAT  38
      ERR(J,K) = ERR(J,K)+EXH(J)                                        STAT  39
    5 VAR(J,J,K) = VAR(J,J,K)+EXH(J)**2                                 STAT  40
C                                                                       STAT  41
C                                                                       STAT  42
      IF (ISTAT.EQ.0) RETURN                                            STAT  43
C                                                                       STAT  44
C                                                                       STAT  45
      DO 6 L=2,N                                                        STAT  46
      LM1 = L-1                                                         STAT  47
C                                                                       STAT  48
```

```
C
      DC 6 J=1,LM1
    6 VAR(L,J,K) = VAR(L,J,K)+EXH(L)*EXH(J)
C
C
      RETURN
      END
```

```
      SLBROUTINE TRACK                                                  TRCK
C     IF TRACK IS TO BE GENERATED ON-LINE IT IS DCNE IN THIS SLBROUTINE TRCK
C     IN THE DEFAULT OPTION (ITRK.EQ.0) THE TRACK IS GENERATED          TRCK
C     FRCM THE STANDARD LINEAR DIFFERENCE EQUATION                      TRCK
C                                                                       TRCK
C     X(K+1)=PHI*X(K)+GAMMA*W(K)                                        TRCK
C                                                                       TRCK
      REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMF2,FKKM1,G,FKK,Q,EI,PKKM10TRCK
     1,MISSD                                                            TRCK
      CCMMCN GAMMA(9,9),COVW(9,9),R(9,9),PHI(9,9),H(9,9),TEMP(9,9),TEMP1TRCK
     1(9,9),TEMP2(9,9),PKKM1(9,9),PKKM10(9,9),G(9,9),PKK(9,9),Q(9,9),EI(TRCK
     29,9),MISSD(100,160),VAR(9,9,160),GKS(9,9,160),PKKS(9,9,160),XM(9,1TRCK
     360),ERR(9,160),GAMMAS(9,9),PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGTRCK
     4W(9),X(9),SIGXZ(9),XZMEAN(9),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),STRCK
     5IGV(9),XHATZ(9),XINT(160),N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXTRCK
     6Z,IV,IW,IEST,ND,NHIT,NSCARE,NMISS,NFIRE                           TRCK
      DIMENSION W(9)                                                    TRCK
C     ITRK NE.0 OR 1 -- SEVERAL TRACKS GENERATED,BUT NOT FROM STD.      TRCK
C                       LINEAR EQS.                                     TRCK
C          = 0 -- SEVERAL TRACKS GENERATED FROM STC LINEAR EQS          TRCK
C          = 1 -- ONLY ONE TRACK IS USED                                TRCK
      IF (ITRK.NE.0) GO TO 4                                            TRCK
      CALL SNCRM (IW,W,IN)                                              TRCK
C     CCNVERT EACH N(0,1) R.V. TO N(0,SIGW(I)) R.V.                     TRCK
C                                                                       TRCK
C                                                                       TRCK
      DC 1 I=1,IN                                                       TRCK
    1 W(I) = SIGW(I)*W(I)                                               TRCK
C                                                                       TRCK
C                                                                       TRCK
C                                                                       TRCK
C                                                                       TRCK
      DC 2 I=1,N                                                        TRCK
    2 X(I) = XS(I,K)                                                    TRCK
C                                                                       TRCK
C                                                                       TRCK
      CALL VPROD (GAMMAS,W,N,IN,W)                                      TRCK
      CALL VPROD (PHIS,X,N,N,VTMP)                                      TRCK
      CALL VADD (VTMP,W,N,VTMP)                                         TRCK
C                                                                       TRCK
C                                                                       TRCK
      DO 3 I=1,N                                                        TRCK
    3 XS(I,K+1) = VTMP(I)                                               TRCK
C                                                                       TRCK
C                                                                       TRCK
C     NEW VALUE OF X HAS BEEN COMPUTED AND STORED IN THE ARRAY XS       TRCK
      RETURN                                                            TRCK
    4 IF (ITRK.NE.1) GO TO 6                                            TRCK
```

```
C        IF(ITRK.EQ.1) THE USER MUST INSERT HERE THE STATEMENTS REQUIRED
C       TO GENERATE A SINGLE TRAJECTORY AND STORE IT IN THE ARRAY
C       XS(I,K),I=1,N,K=2,NSAM (NOTE THAT IF A SINGLE TRAJECTORY IS TO BE
C       GENERATED, THE INITIAL CONDITION HAS BEEN READ IN AND STORED
C       IN XS(I,1),I=1,N)
C
C
        DO 5 K=2,NSAM
        EKM1 = K-1
        T = 0.5*EKM1
        XS(1,K) = 13.-0.0173205*T
        XS(2,K) = -0.0173205
        XS(3,K) = 7.7-0.01*T
      5 XS(4,K) = -0.01
C
C
        RETURN
      6 CONTINUE
C       IF THIS POINT IS REACHED, ITRK NOT EQUAL 0 OR 1 INDICATING THAT
C       SEVERAL TRACKS ARE TO BE GENERATED, BUT NOT BY USING THE STD.
C       LINEAR DIFFERENCE EQS..THE USER MUST SUPPLY THE APPROPRIATE
C       STATEMENTS HERE.
        RETURN
        END
```

```
      SLBROUTINE XZERO                                                  XZER
C     THIS SUBROUTINE GENERATES THE INITIAL STATE VALUE FRCM A NORMAL   XZER
C     RANDCM NUMBER GENERATOR. IT IS ASSUMED THAT THE INITIAL STATE     XZER
C     HAS COMPONENTS THAT ARE INDEPENDENT                               XZER
      REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI,PKKM10XZER
     1,MISSD                                                            XZER
      CCMMON GAMMA(9,9),COVW(9,9),R(9,9),PHI(9,9),H(9,9),TEMP(9,9),TEMP1XZER
     1(9,9),TEMP2(9,9),PKKM1(9,9),PKKM10(9,9),G(9,9),PKK(9,9),Q(9,9),EI(XZER
     29,9),MISSD(100,160),VAR(9,9,160),GKS(9,9,160),PKKS(9,9,160),XM(9,1XZER
     360),ERR(9,160),GAMMAS(9,9),PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGXZER
     4W(9),X(9),SIGXZ(9),XZMEAN(9),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SXZER
     5IGV(9),XHATZ(9),XINT(160),N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXXZER
     6Z,IV,IW,IEST,ND,NHIT,NSCARE,NMISS,NFIRE                           XZER
      CALL SNCRM (IXZ,X,N)                                              XZER
C                                                                       XZER
C                                                                       XZER
      DC 1 I=1,N                                                        XZER
    1 XS(I,1) = SIGXZ(I)*X(I)+XZMEAN(I)                                 XZER
C                                                                       XZER
C                                                                       XZER
      RETURN                                                            XZER
      END                                                               XZER
```

```
      SUBROUTINE ADD (A,B,N,M,C)                                        MADD   1
C     THIS SUBROUTINE ADDS THE NXM MATRICES A AND B, STORING THE        MADD   2
C     RESULT IN C                                                       MADD   3
      REAL *8A,B,C                                                      MADD   4
      DIMENSION A(9,9), B(9,9), C(9,9)                                  MADD   5
C                                                                       MADD   6
C                                                                       MADD   7
      DO 1 I=1,N                                                        MADD   8
C                                                                       MADD   9
C                                                                       MADD  10
      DO 1 J=1,M                                                        MADD  11
    1 C(I,J) = A(I,J)+B(I,J)                                            MADD  12
C                                                                       MADD  13
C                                                                       MADD  14
      RETURN                                                            MADD  15
      END                                                               MADD  16
```

```
      SUBROUTINE MREAD (A,N,M)
C     THIS SUBROUTINE READS AN NXM MATRIX A ACCORDING TO THE FORMAT
C     8F10.0. THE ENTRIES IN THE FIRST ROW OF A ARE READ FIRST, THEN
C     THE ENTRIES IN THE SECOND ROW, ANC SO ON.
      REAL *8A
      DIMENSION A(9,9)
C
C
      DC 1 I=1,N
    1 READ (5,2) (A(I,J),J=1,M)
C
C
      RETURN
C
C
    2 FCRMAT (8F10.0)
      END
```

```
      SUBROUTINE MWRITE (A,N,M)                                         MWRT   1
C     THIS SUBROUTINE WRITES THE ENTRIES OF THE NXM MATRIX A            MWRT   2
      REAL *8A                                                          MWRT   3
      DIMENSION A(9,9)                                                  MWRT   4
C                                                                       MWRT   5
C                                                                       MWRT   6
      DO 1 I=1,N                                                        MWRT   7
    1 WRITE (6,2) (A(I,J),J=1,M)                                        MWRT   8
C                                                                       MWRT   9
C                                                                       MWRT  10
      RETURN                                                            MWRT  11
C                                                                       MWRT  12
C                                                                       MWRT  13
    2 FORMAT (9(2X,1PE12.5))                                            MWRT  14
      END                                                               MWRT  15
```

```
      SUBROUTINE PROD (A,B,N,M,L,C)                                     MPRD   1
C     THIS SUBROUTINE COMPUTES THE MATRIX PRODUCT AB AND STORES THE     MPRD   2
C     RESULT IN C                                                       MPRD   3
C     A = NXM,   B = MXL,   C = NXL                                     MPRD   4
      REAL *8A,B,C,T                                                    MPRD   5
      DIMENSICN A(9,9), B(9,9), C(9,9), T(9,9)                          MPRD   6
C                                                                       MPRD   7
C                                                                       MPRD   8
      DO 1 I=1,N                                                        MPRD   9
C                                                                       MPRD  10
C                                                                       MPRD  11
      DC 1 J=1,L                                                        MPRD  12
    1 T(I,J)  = 0.0                                                     MPRD  13
C                                                                       MPRD  14
C                                                                       MPRD  15
C                                                                       MPRD  16
C                                                                       MPRD  17
      DC 2 I=1,N                                                        MPRD  18
C                                                                       MPRD  19
C                                                                       MPRD  20
      DO 2 J=1,L                                                        MPRD  21
C                                                                       MPRD  22
C                                                                       MPRD  23
      DC 2 K=1,M                                                        MPRD  24
    2 T(I,J)  = T(I,J)+A(I,K)*B(K,J)                                    MPRD  25
C                                                                       MPRD  26
C                                                                       MPRD  27
C                                                                       MPRD  28
C                                                                       MPRD  29
      DC 3 I=1,N                                                        MPRD  30
C                                                                       MPRD  31
C                                                                       MPRD  32
      DO 3 J=1,L                                                        MPRD  33
    3 C(I,J)  = T(I,J)                                                  MPRD  34
C                                                                       MPRD  35
C                                                                       MPRD  36
      RETURN                                                            MPRD  37
      END                                                               MPRD  38
```

```
      SUBROUTINE SUB (A,B,N,M,C)
C     THIS SUBROUTINE SUBTRACTS THE NXM MATRIX B FROM THE NXM MATRIX
C     A AND STORES THE RESULT IN C
      REAL *8A,B,C
      DIMENSION A(9,9), B(9,9), C(9,9)
C
C
      DC 1 I=1,N
C
C
      DC 1 J=1,M
    1 C(I,J) = A(I,J)-B(I,J)
C
C
      RETURN
      END
```

```
      SUBROUTINE TRANS (A,N,M,C)
C     THIS SUBROUTINE FORMS THE MATRIX TRANSPOSE CF A STORING THE
C     RESULT IN C
C     A = NXM,    C = MXN
      REAL *8A,C
      DIMENSICN A(9,9), C(9,9)
C
C
      DC 1 I=1,N
C
C
      DC 1 J=1,M
    1 C(J,I) = A(I,J)
C
C
      RETURN
      END
```

```
      SUBROUTINE VADD (X,Y,N,Z)                                         VADD   1
C     THIS SUBROUTINE COMPUTES THE SUM OF THE N-VECTORS X AND           VADD   2
C     Y AND STORES THE RESULT IN THE N-VECTOR Z                         VADD   3
C                                                                       VADD   4
      REAL *4X(9),Y(9),Z(9)                                             VADD   5
C                                                                       VADD   6
C                                                                       VADD   7
      DC 1 I=1,N                                                        VADD   8
    1 Z(I) = X(I)+Y(I)                                                  VADD   9
C                                                                       VADD  10
C                                                                       VADD  11
      RETURN                                                            VADD  12
      END                                                               VADD  13
```

```
      SUBROUTINE VPROD (A,X,M,N,Y)
C     THIS SUBROUTINE COMPUTES THE PRODUCT OF THE MXN MATRIX
C     A AND THE N-VECTOR X AND STORES THE RESULT IN THE
C     M-VECTOR Y
C
C
      REAL *4A(9,9),X(9),Y(9),T(9)
C
C
      DO 1 I=1,M
      T(I) = 0.DO
C
C
      DC 1 J=1,N
    1 T(I) = T(I)+A(I,J)*X(J)
C
C
C
C
      DC 2 I=1,M
    2 Y(I) = T(I)
C
C
      RETURN
      END
```

```
      SUBROUTINE VREAD (V,N)                                              VRED   1
C                                                                         VRED   2
C     THIS SUBROUTINE READS THE N-DIMENSIONAL S.P. VECTOR V               VRED   3
C                                                                         VRED   4
      DIMENSION V(9)                                                      VRED   5
      READ (5,1) (V(I),I=1,N)                                             VRED   6
      RETURN                                                              VRED   7
C                                                                         VRED   8
C                                                                         VRED   9
    1 FORMAT (8F10.0)                                                     VRED  10
      END                                                                 VRED  11
```

```
      SUBROUTINE VSUB (X,Y,N,Z)                                         VSUB   1
C     THIS SUBROUTINE COMPUTES THE DIFFERENCE X-Y OF THE TWO            VSUB   2
C     N-VECTORS X & Y AND STORES THE RESULT IN THE N-VECTOR Z           VSUB   3
C                                                                       VSUB   4
      REAL *4X(9),Y(9),Z(9)                                             VSUB   5
C                                                                       VSUB   6
C                                                                       VSUB   7
      DO 1 I=1,N                                                        VSUB   8
    1 Z(I) = X(I)-Y(I)                                                  VSUB   9
C                                                                       VSUB  10
C                                                                       VSUB  11
      RETURN                                                            VSUB  12
      END                                                               VSUB  13
```

```
      SLBROUTINE VWRITE (V,N)
C
C     THIS SUBROUTINE WRITES THE N-DIMENSIONAL S.P. VECTOR V
C
      DIMENSION V(9)
      WRITE (6,1) (V(I),I=1,N)
      RETURN
C
C
    1 FCRMAT (9(2X,1PE12.5))
      END
```

A.2 LISTING OF KEY VARIABLES USED IN THE MONTE CARLO PROGRAM

AMISS	average number of misses over the entire track (scalar).
COVW(m,m)	covariance of the system random forcing input matrix (double precision).
EI(n,n)	identity matrix (double precision).
ERR(N,nsam)	sample mean of estimation error vector (single precision).
G(n,m)	sample gain matrix (double precision).
GAMMA(n,m)	system random forcing input distribution matrix (double precision).
GAMMAS(n,m)	system random forcing input distribution matrix (single precision).
GK(n,m)	sample gain matrix (single precision).
GKS(n,m,nsam)	filter gain schedule (single precision).
H(m,n)	system observation matrix (double precision).
HIT	average number of hits over the entire track (scalar).
HS(m,n)	system observation matrix (single precision).
ITER	ensemble member index.
M	number of measurements.
MISSD(nens,nsam)	sample miss distance matrix (double precision).
N	system order.
NENS	number of members in the ensemble.
NSAM	number of time points to be processed.
PHI(n,n)	state transition matrix (double precision).

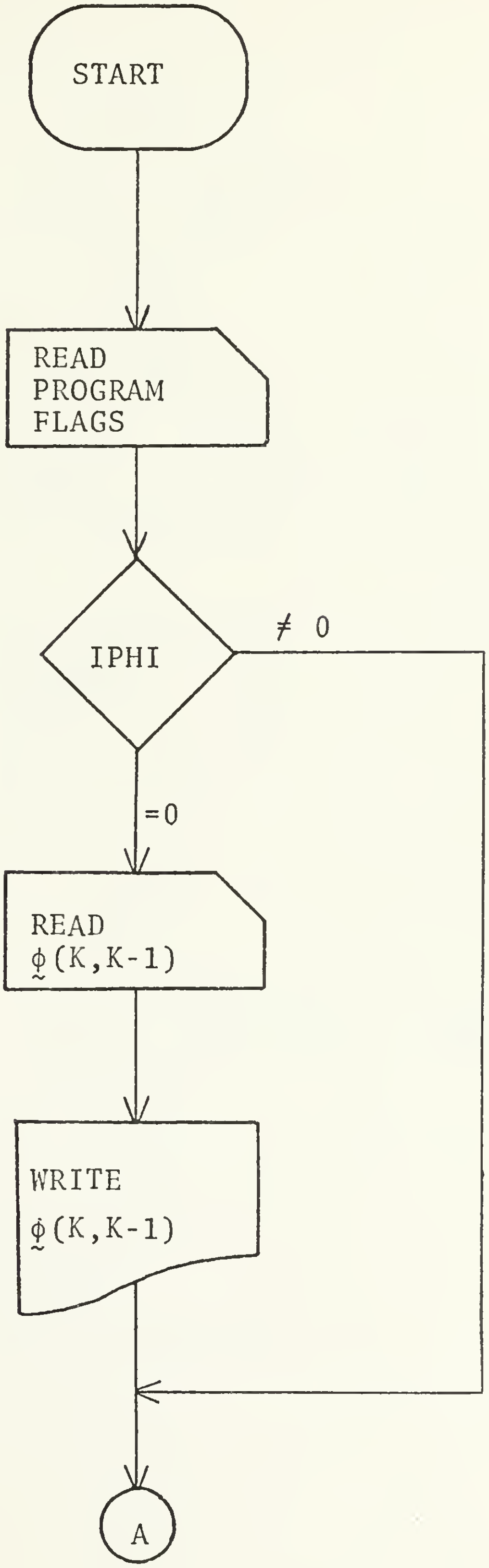
PHIS(n,n)	state transition matrix (single precision).
PKK(n,n)	theoretical covariance of estimation error matrix $\underset{\sim}{P}(K/K)$ (double precision).
PKKM1(n,n)	one-step prediction error covariance matrix $\underset{\sim}{P}(K+1/K)$ (double precision).
PKKM10(n,n)	initial value of the covariance of estimation error matrix $\underset{\sim}{P}(0/-1)$ (double precision).
PKKS(n,n,nsam)	sample covariance of estimation error matrix $\underset{\sim}{P}(K/K)$ (single precision).
Q(m,m)	system random forcing input covariance matrix (double precision).
R(m,m)	covariance of measurement noise matrix (double precision).
SCARE	average number of scares over the entire track.
SIGV(m)	standard deviation of measurement noise vector (single precision).
SIGW(m)	standard deviation of system random forcing input vector (single precision).
SIGXZ(n)	standard deviation of system initial condition vector (single precision).
V(m)	measurement noise vector (single precision).
VAR(n,n,nsam)	actual covariance of estimation error matrix (single precision).
W(m)	system random forcing input vector (single precision).
X(n)	state vector (single precision).
XHATZ(n)	initializing value of the state estimate vector $\hat{\underset{\sim}{x}}(0/-1)$ (single precision).
XHKK(n)	state estimate vector $\hat{\underset{\sim}{x}}(K/K)$ (single precision).
XHKKM1(n)	one-step state prediction vector $\hat{\underset{\sim}{x}}(K/K-1)$ (single precision).

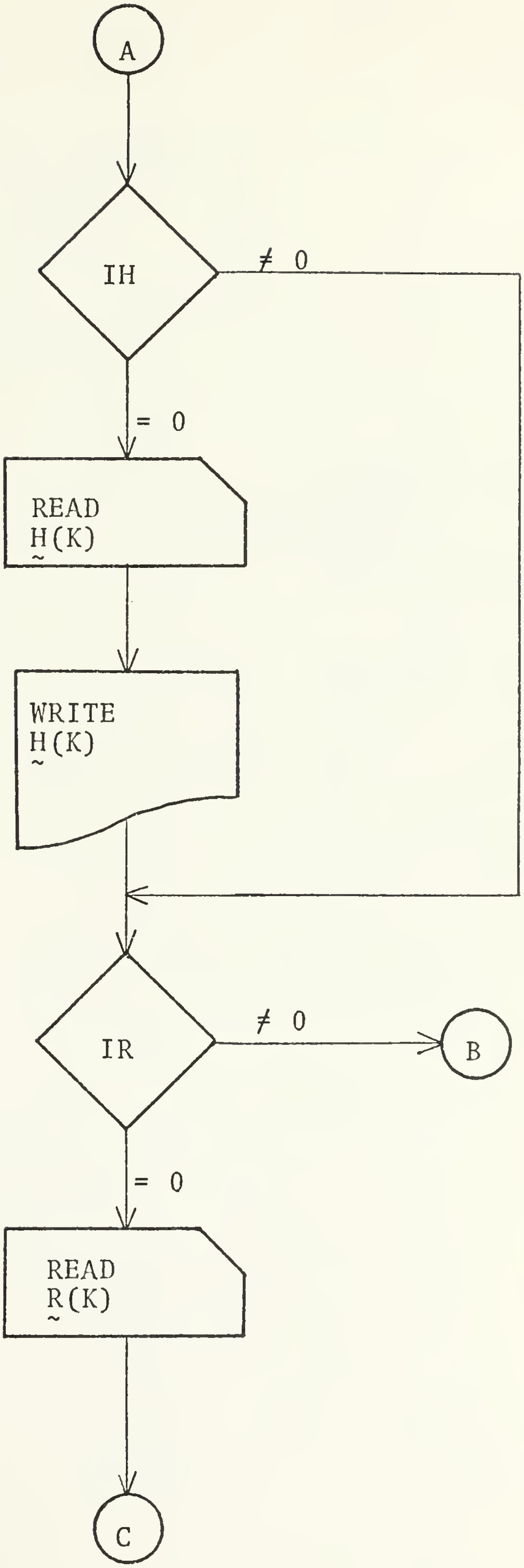
XM(n,nsam)	mean state trajectory over the ensemble (single precision).
XS(n,nsam)	reference trajectory (single precision).
XZMEAN(n)	mean of initial condition vector (single precision).

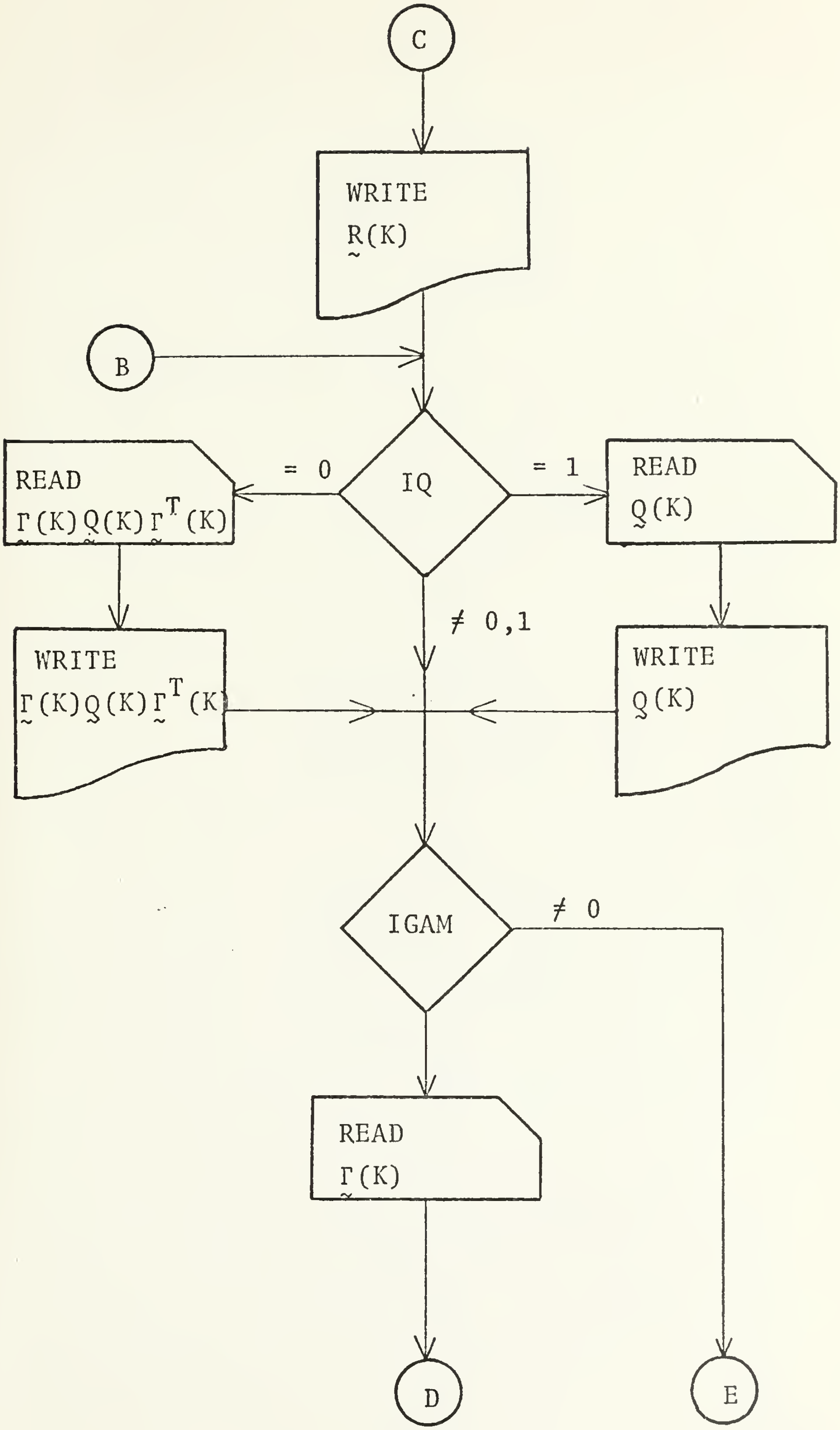
A.3 IBM SYSTEM 360/67 SOURCE LIBRARY SUBROUTINES CALLED BY THE MONTE CARLO SIMULATION PROGRAM

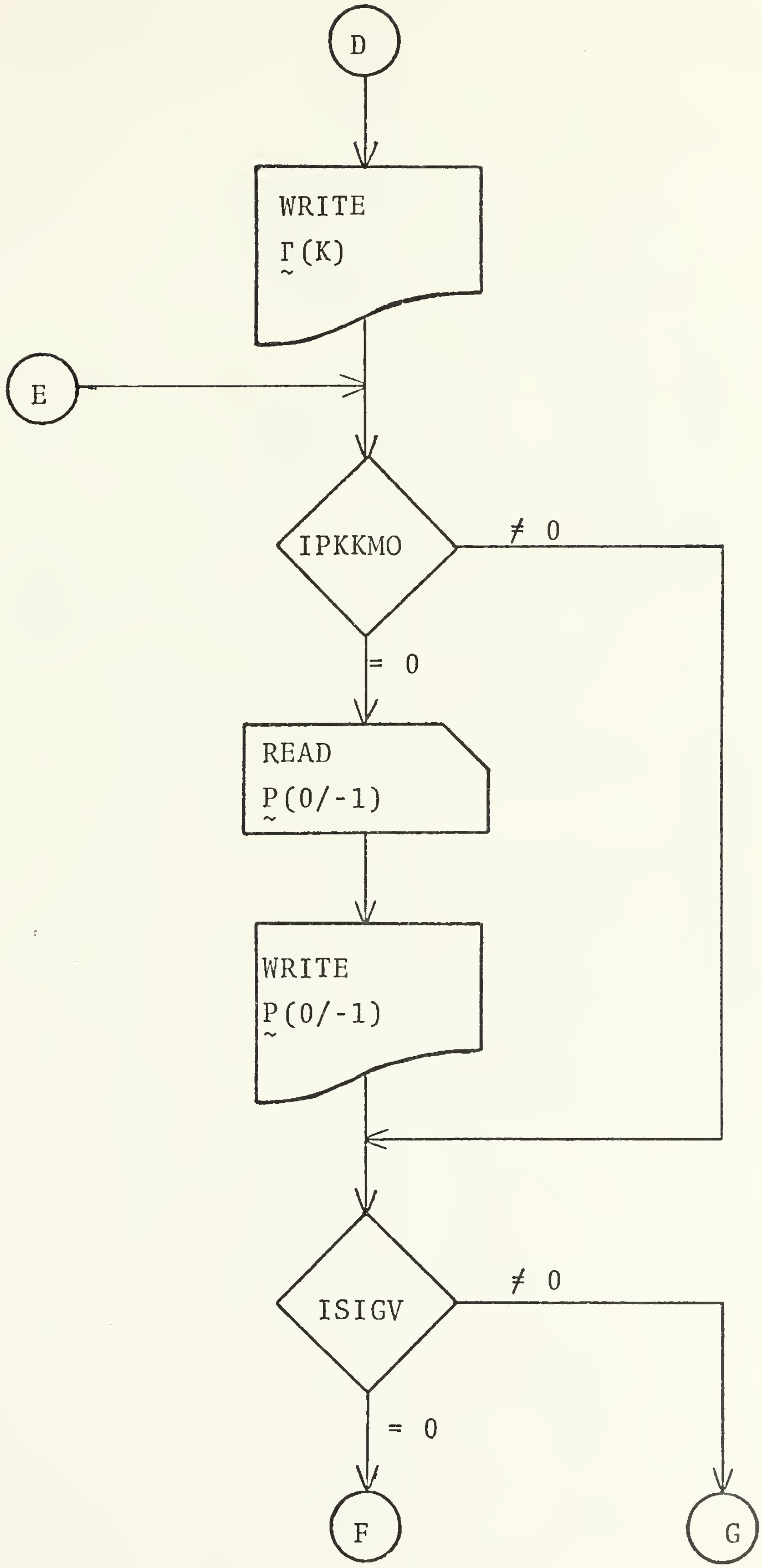
GAUSS3	matrix inversion by the method of GAUSS.
OVFLOW	utility program required by the random number generator.
PLOTP	generates two-dimensional line printer plots.
SNORM	random number generator that gives normally distributed, shuffled pseudo-random numbers.

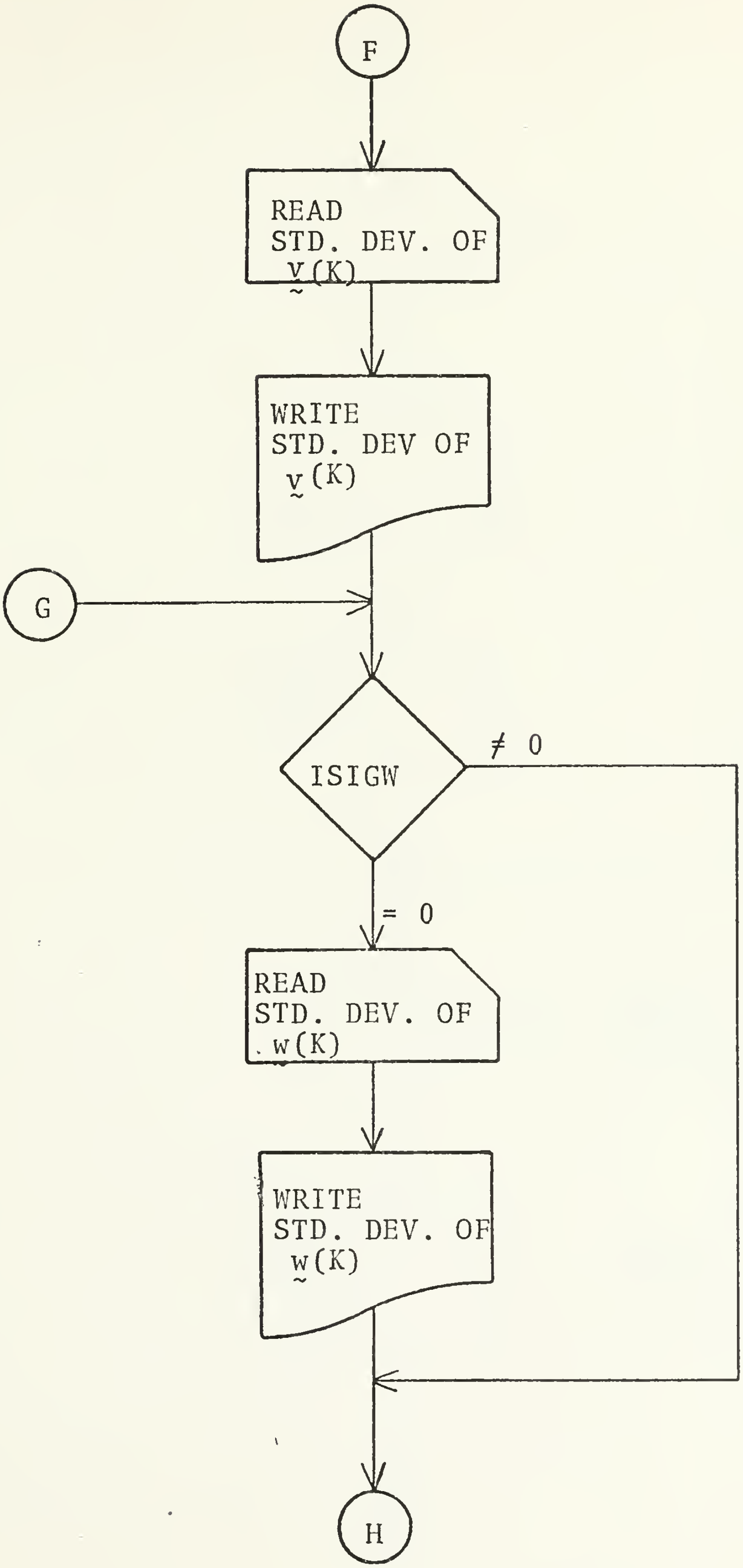
A.4 MONTE CARLO SIMULATION PROGRAM FLOWCHART

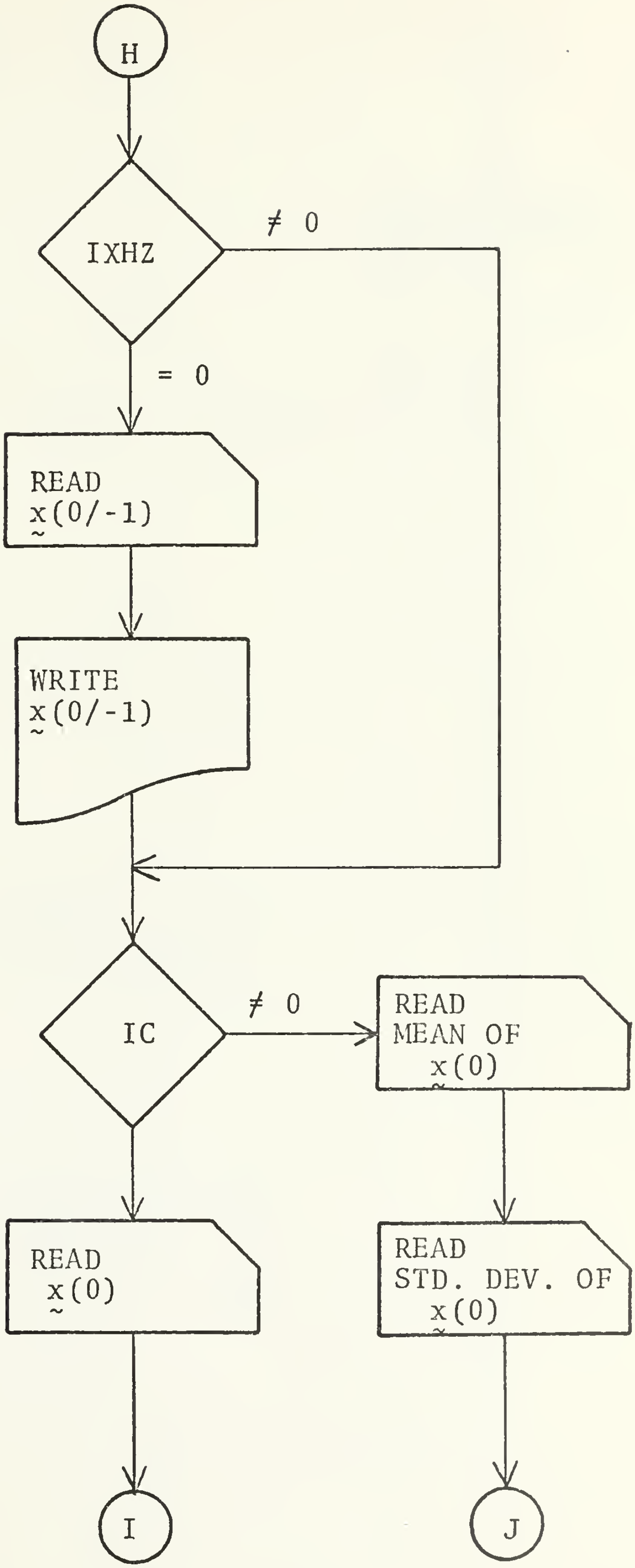

START
READ PROGRAM FLAGS
IPHI
≠ 0
=0
READ $\underset{\sim}{\phi}(K,K-1)$
WRITE $\underset{\sim}{\phi}(K,K-1)$
A

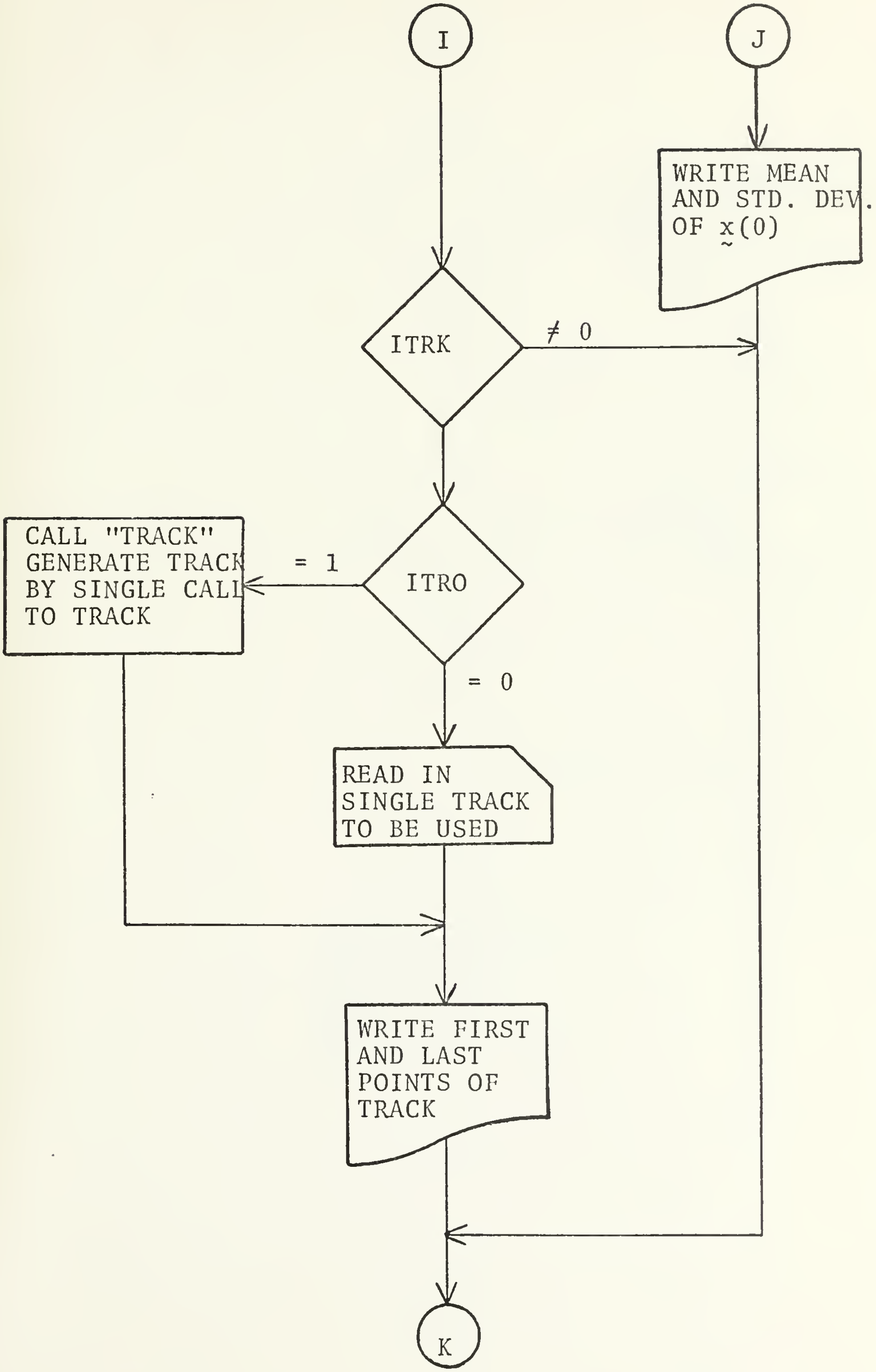

A
IH
≠ 0
= 0
READ
H(K)
~
WRITE
H(K)
~
IR
≠ 0
B
= 0
READ
R(K)
~
C

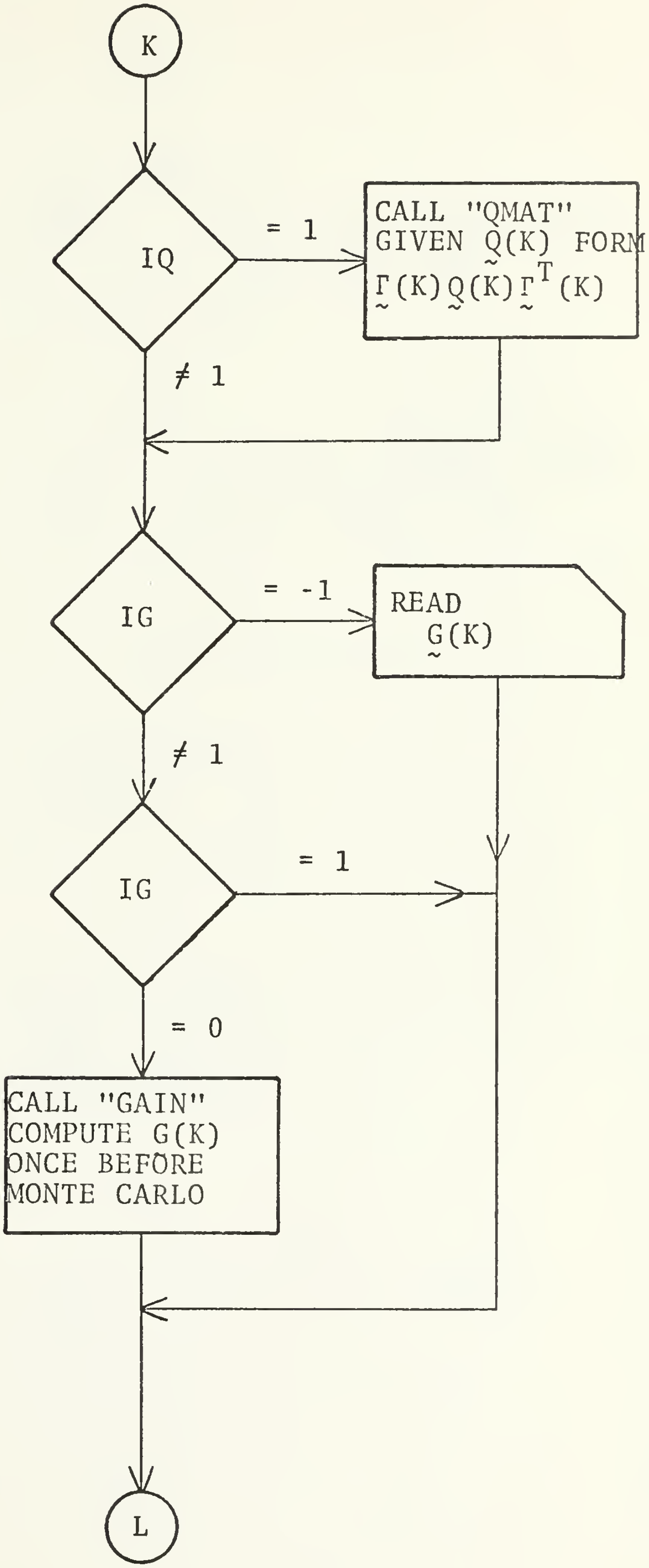

C
WRITE
R(K)
B
IQ
= 0
= 1
READ
Γ(K) Q(K) Γ^T(K)
READ
Q(K)
≠ 0,1
WRITE
Γ(K) Q(K) Γ^T(K)
WRITE
Q(K)
IGAM
≠ 0
READ
Γ(K)
D
E

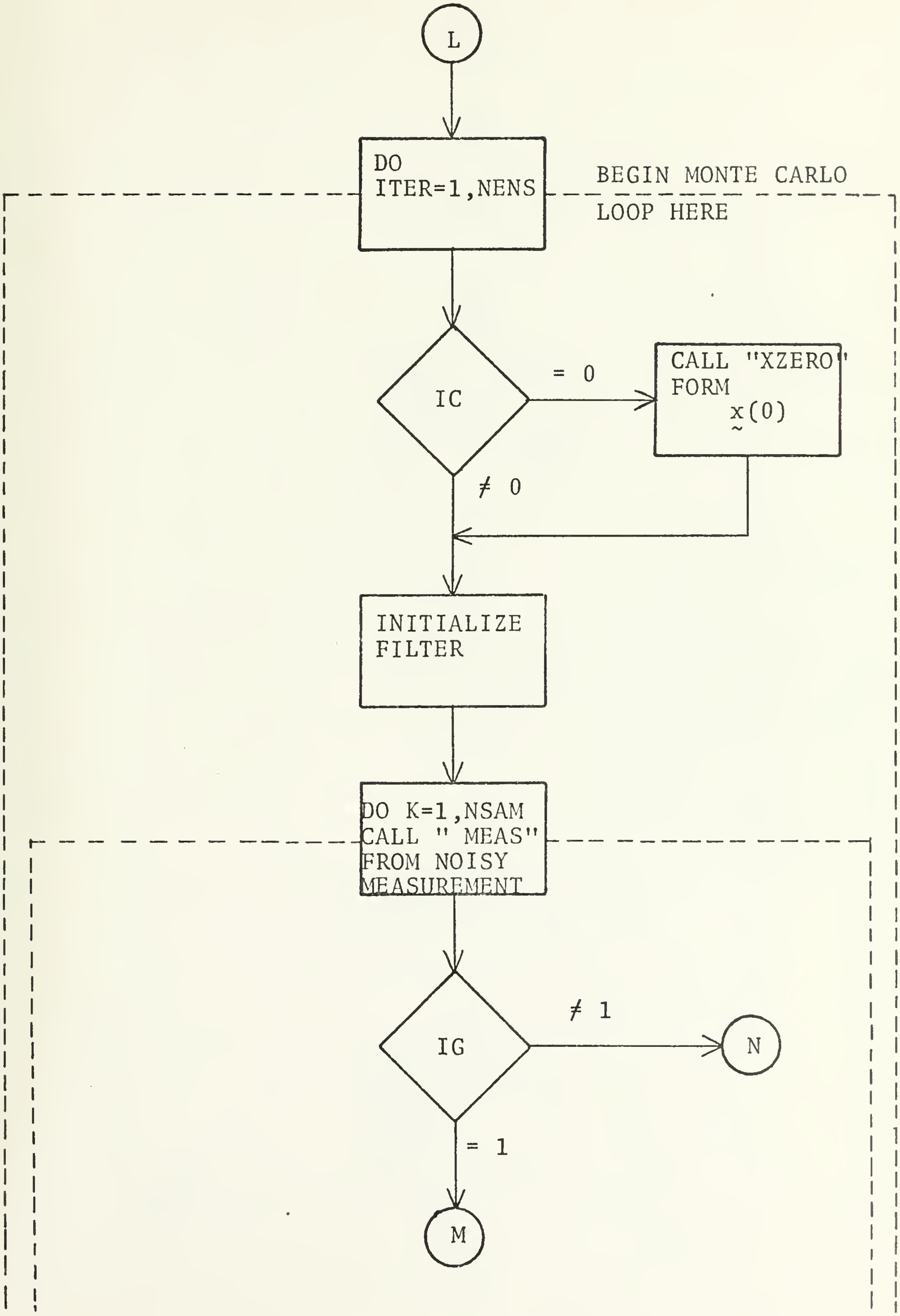

D
WRITE
Γ(K)
E
IPKKMO
≠ 0
= 0
READ
P(0/-1)
WRITE
P(0/-1)
ISIGV
≠ 0
= 0
F
G

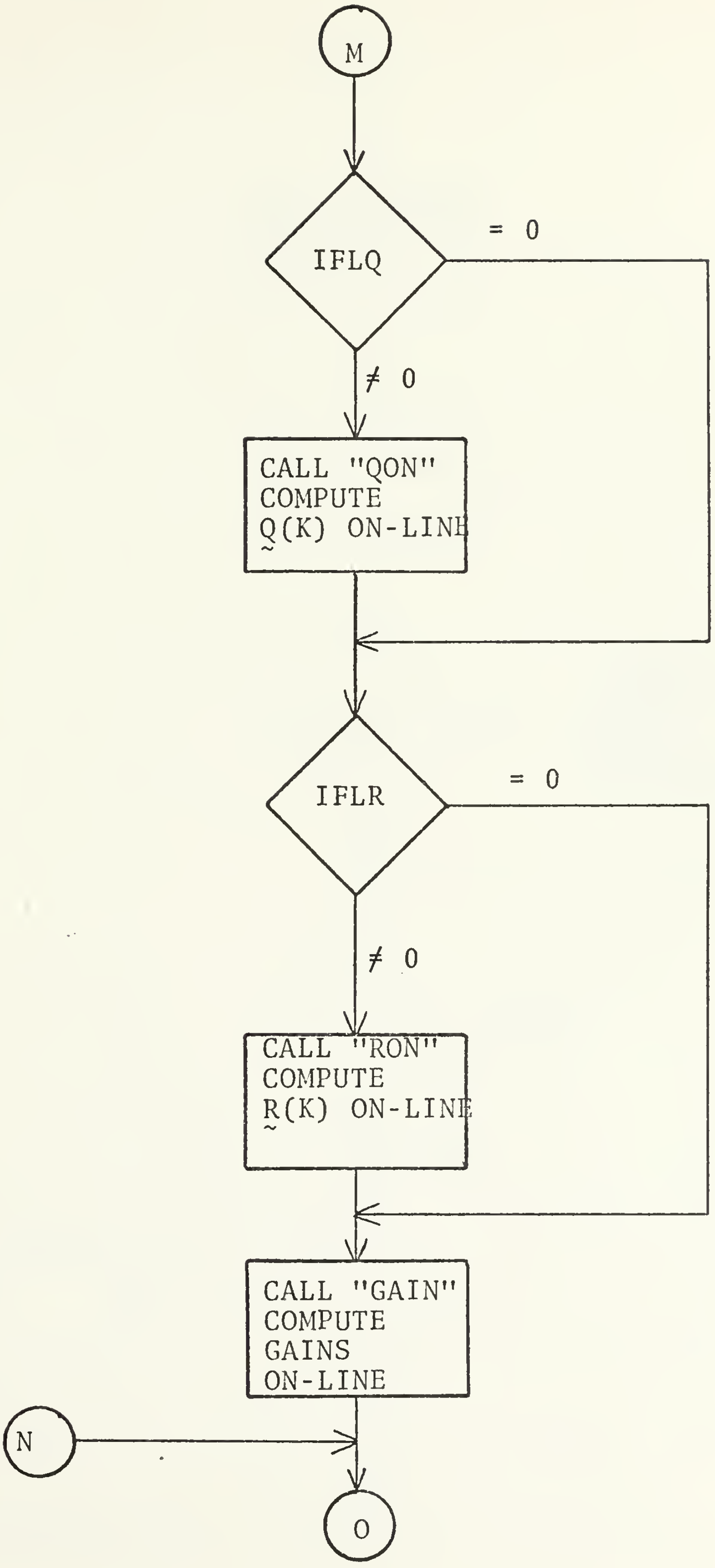

F
READ
STD. DEV. OF
v(K)
WRITE
STD. DEV OF
v(K)
G
ISIGW
≠ 0
= 0
READ
STD. DEV. OF
w(K)
WRITE
STD. DEV. OF
w(K)
H

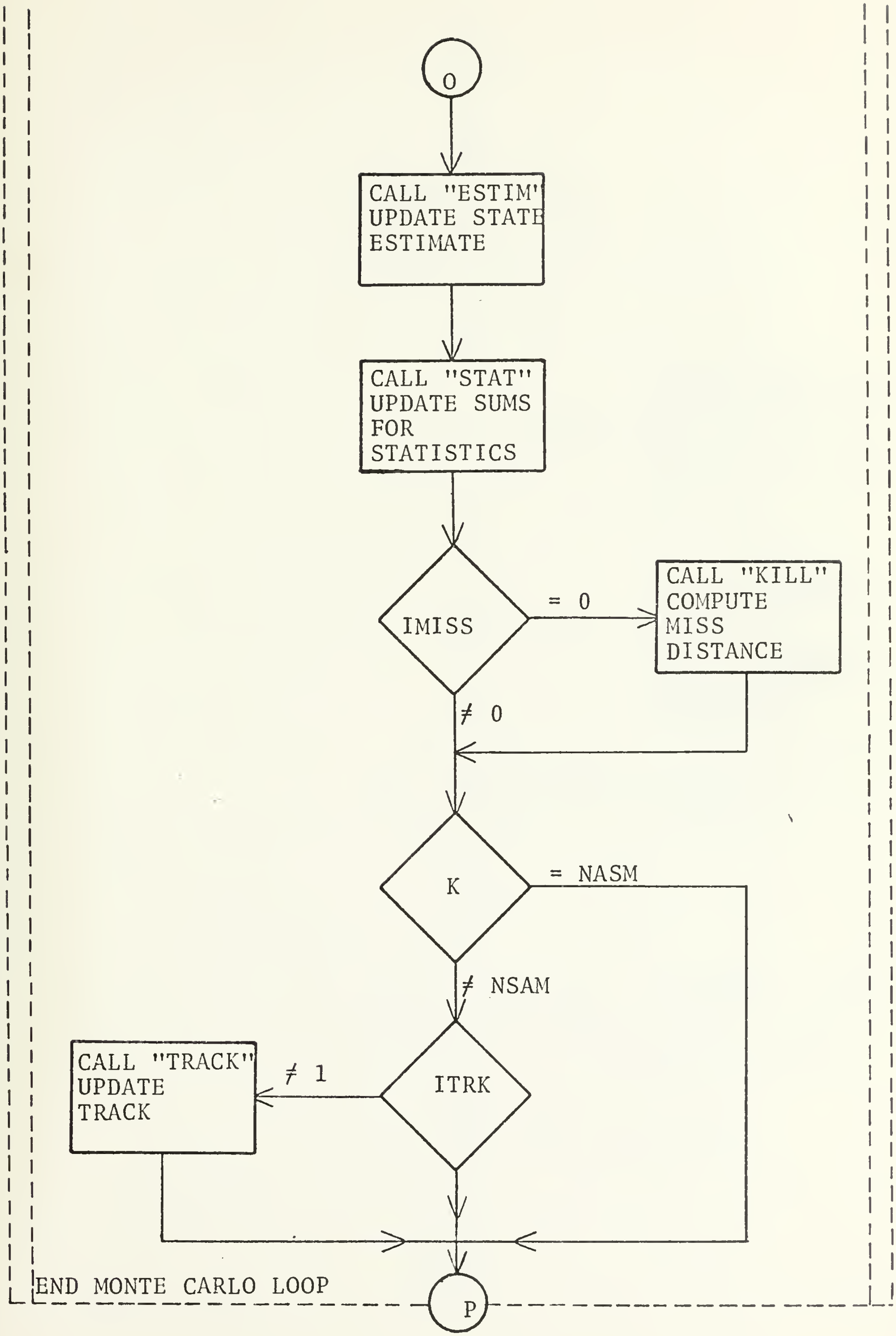

H
IXHZ
≠ 0
= 0
READ
x(0/-1)
WRITE
x(0/-1)
IC
≠ 0
READ
MEAN OF
x(0)
READ
x(0)
READ
STD. DEV. OF
x(0)
I
J

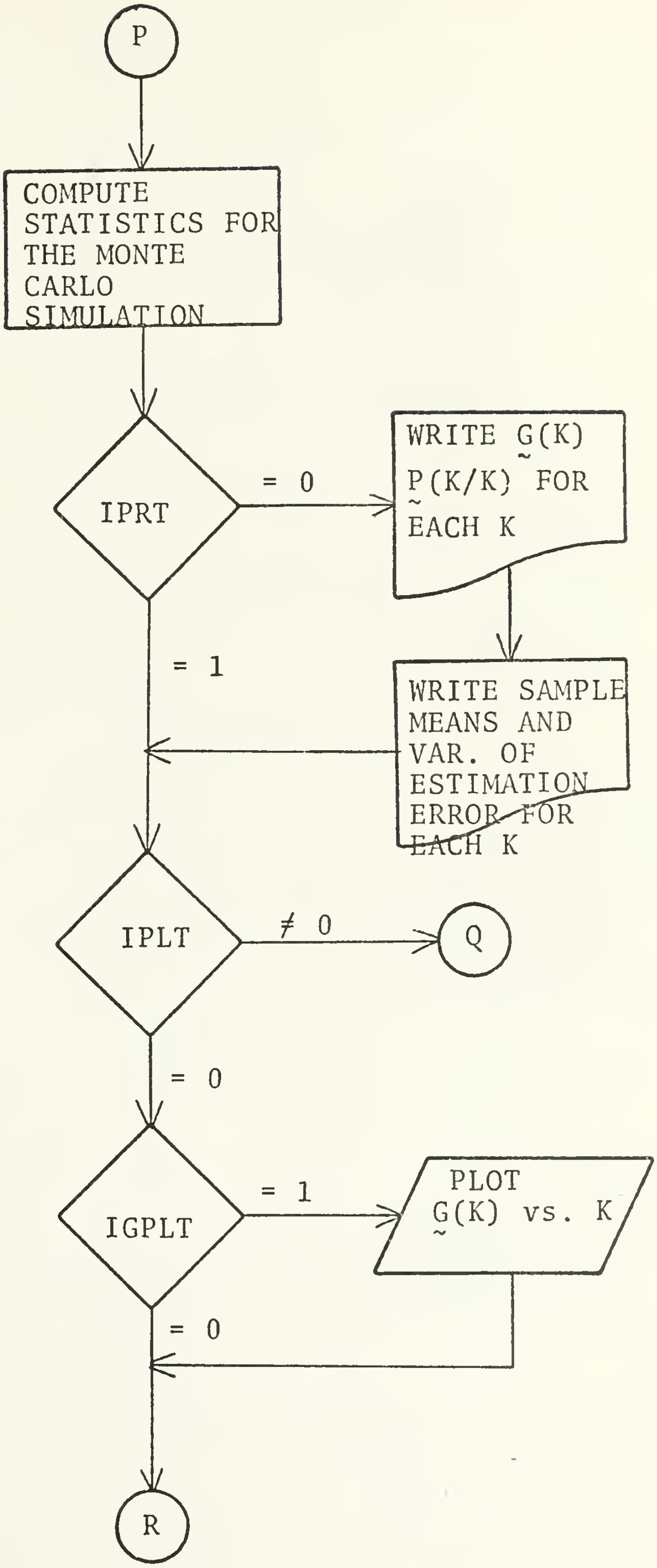

I
J
WRITE MEAN AND STD. DEV. OF $\underset{\sim}{x}(0)$
ITRK
≠ 0
ITRO
= 1
CALL "TRACK" GENERATE TRACK BY SINGLE CALL TO TRACK
= 0
READ IN SINGLE TRACK TO BE USED
WRITE FIRST AND LAST POINTS OF TRACK
K

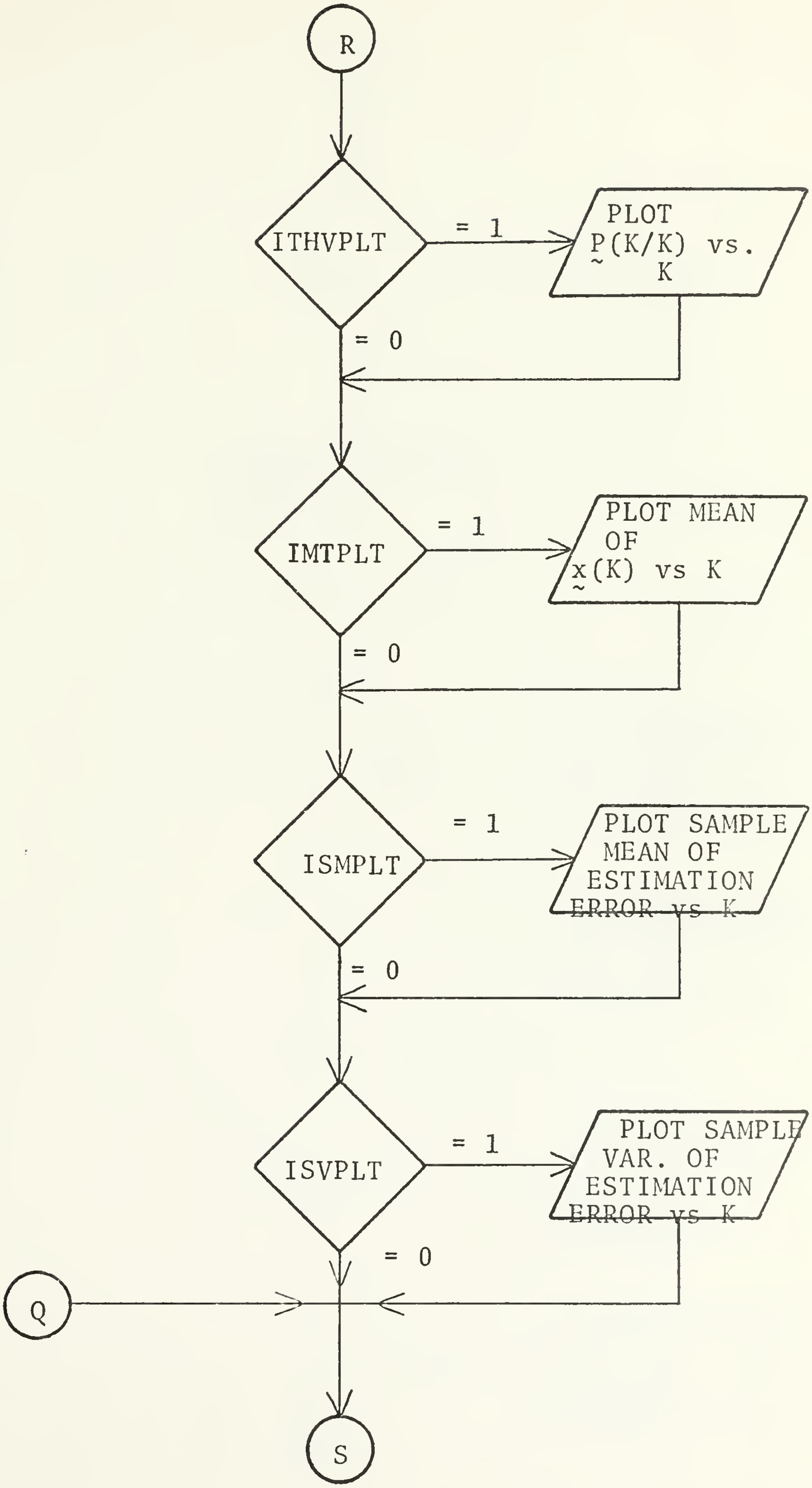

K
IQ
= 1
CALL "QMAT"
GIVEN Q(K) FORM
Γ(K)Q(K)Γ^T(K)
≠ 1
IG
= -1
READ
G(K)
≠ 1
IG
= 1
= 0
CALL "GAIN"
COMPUTE G(K)
ONCE BEFORE
MONTE CARLO
L

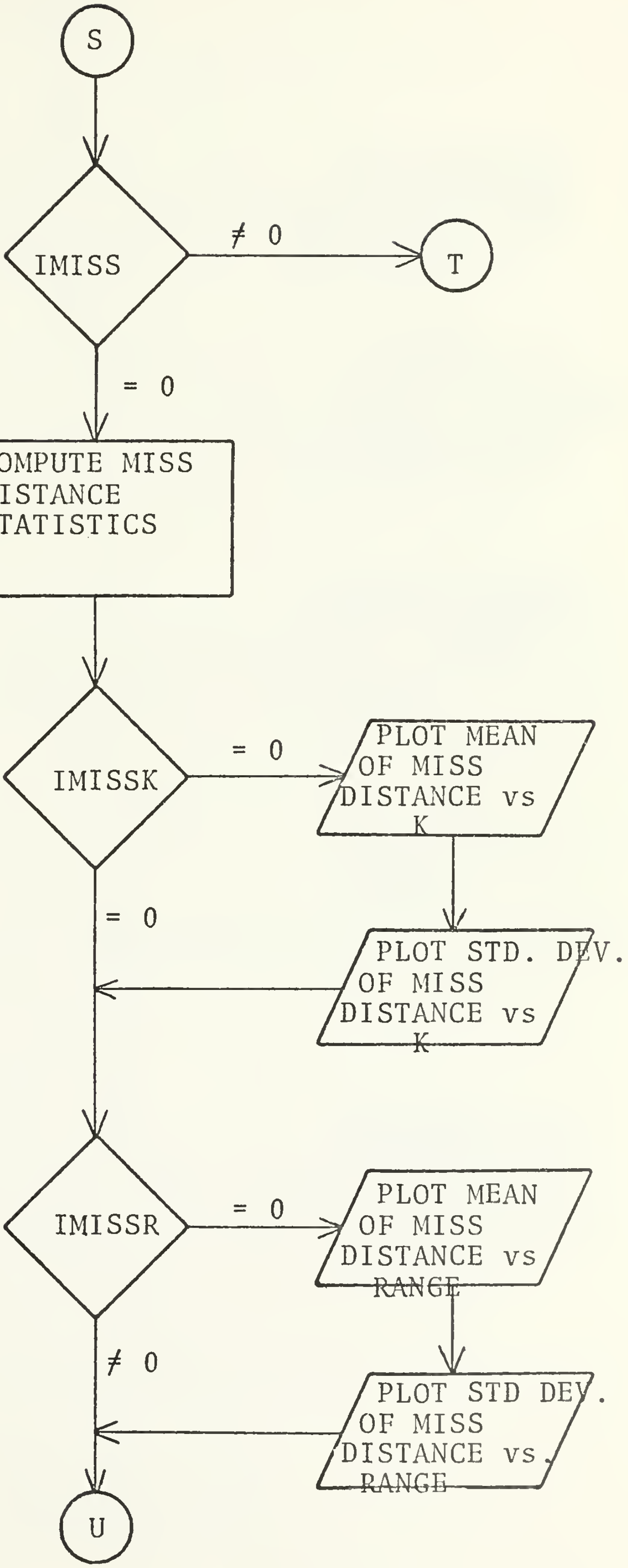

L
DO
ITER=1,NENS
BEGIN MONTE CARLO
LOOP HERE
IC
= 0
CALL "XZERO"
FORM
$\underset{\sim}{x}(0)$
≠ 0
INITIALIZE
FILTER
DO K=1,NSAM
CALL " MEAS"
FROM NOISY
MEASUREMENT
IG
≠ 1
N
= 1
M

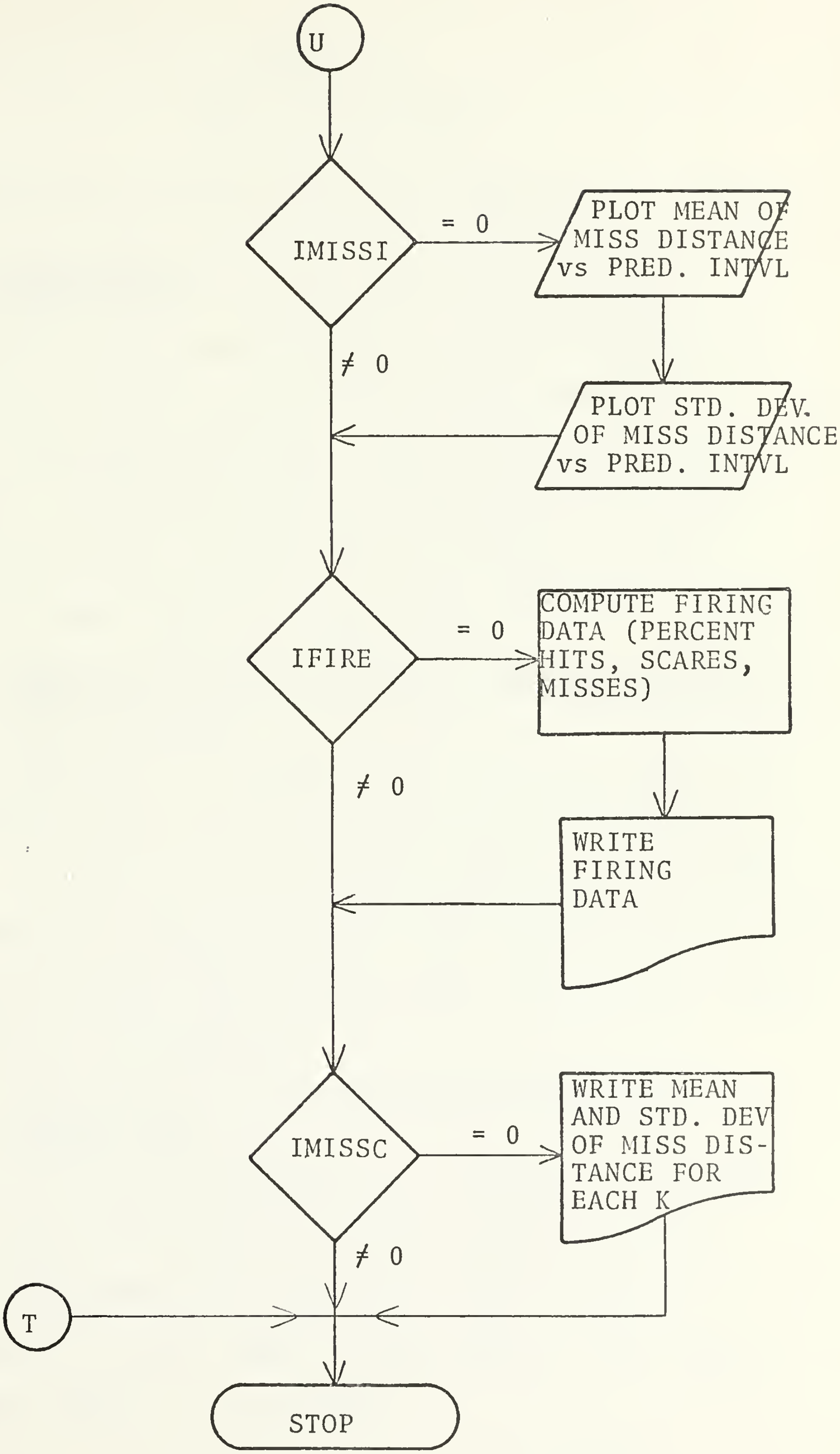

M
IFLQ
= 0
≠ 0
CALL "QON"
COMPUTE
Q(K) ON-LINE
IFLR
= 0
≠ 0
CALL "RON"
COMPUTE
R(K) ON-LINE
CALL "GAIN"
COMPUTE
GAINS
ON-LINE
N
O


O
CALL "ESTIM" UPDATE STATE ESTIMATE
CALL "STAT" UPDATE SUMS FOR STATISTICS
IMISS
= 0
CALL "KILL" COMPUTE MISS DISTANCE
≠ 0
K
= NASM
≠ NSAM
ITRK
≠ 1
CALL "TRACK" UPDATE TRACK
END MONTE CARLO LOOP
P


P
COMPUTE STATISTICS FOR THE MONTE CARLO SIMULATION
IPRT
= 0
WRITE G(K) P(K/K) FOR EACH K
= 1
WRITE SAMPLE MEANS AND VAR. OF ESTIMATION ERROR FOR EACH K
IPLT
≠ 0
Q
= 0
IGPLT
= 1
PLOT G(K) vs. K
= 0
R


R
ITHVPLT
= 1
PLOT P(K/K) vs. K
= 0
IMTPLT
= 1
PLOT MEAN OF x(K) vs K
= 0
ISMPLT
= 1
PLOT SAMPLE MEAN OF ESTIMATION ERROR vs K
= 0
ISVPLT
= 1
PLOT SAMPLE VAR. OF ESTIMATION ERROR vs K
= 0
Q
S


S
IMISS
≠ 0
T
= 0
COMPUTE MISS DISTANCE STATISTICS
IMISSK
= 0
PLOT MEAN OF MISS DISTANCE vs K
= 0
PLOT STD. DEV. OF MISS DISTANCE vs K
IMISSR
= 0
PLOT MEAN OF MISS DISTANCE vs RANGE
≠ 0
PLOT STD DEV. OF MISS DISTANCE vs. RANGE
U


U
IMISSI
= 0
PLOT MEAN OF MISS DISTANCE vs PRED. INTVL
≠ 0
PLOT STD. DEV. OF MISS DISTANCE vs PRED. INTVL
IFIRE
= 0
COMPUTE FIRING DATA (PERCENT HITS, SCARES, MISSES)
≠ 0
WRITE FIRING DATA
IMISSC
= 0
WRITE MEAN AND STD. DEV OF MISS DISTANCE FOR EACH K
≠ 0
T
STOP

APPENDIX B

TEST TRAJECTORIES

B.1 DESCRIPTION OF TEST TRAJECTORY PROFILES

a. Trajectory 1; short-range, low speed, non-maneuvering

Trajectory 1 commences at a range of 3850 yards in a 30 degree dive with an initial velocity of -230 yds/sec (the negative velocity indicates motion toward the origin). The trajectory is terminated at a final range of 457 yards. No maneuver is executed. Sixty data points were taken at a 4-Hz rate.

b. Trajectory 2; short-range, low speed, moderate level maneuver

Trajectory 2 commences at a range of 3850 yards in a 30 degree dive with an initial velocity of -230 yds/sec. At 3450 yards the target executes a 2.0 g $\sin^2$ planar maneuver lasting 13.5 sec. The maneuver is completed at the final sample range of 465 yards. Sixty data points were taken at a 4-Hz rate.

c. Trajectory 3; long-range, low speed, non-maneuvering

Trajectory 3 commences at a range of 9225 yards in a 30 degree dive with an initial velocity of -230 yds/sec. No maneuver is executed. The trajectory is terminated at a final range of 5832 yards. Sixty data points were taken at a 4-Hz rate.

d. Trajectory 4; long-range, low speed, moderate level maneuver

Trajectory 4 commences at a range of 9225 yards in a 30 degree dive with an initial velocity of -230 yds/sec. At 8822 yards, the target executes a 2.0 g $\sin^2$ planar maneuver lasting 13.5 sec. The maneuver is completed at the final sample range of 5840 yards. Sixty data points were taken at a 4-Hz rate.

e. Trajectory 5; medium-range, high speed, evasive level maneuver

Trajectory 5 commences at a range of 4500 yards in a 20 degree dive with an initial velocity of -300 yds/sec. At 3900 yards the target executes a 6.0 g $\sin^2$ planar maneuver lasting 10 sec. The maneuver is completed at a range of 914 yards and the trajectory is terminated at a final range of 90 yards. Sixty data points were taken at a 4-Hz rate.

f. Trajectory 6; long-range, high speed, evasive level maneuver

Trajectory 6 commences at a range of 8000 yards in a 20 degree dive with an initial velocity of -300 yds/sec. At 7500 yards the target executes a 6.0 g $\sin^2$ planar maneuver lasting 10 sec. The maneuver is completed at a range of 4340 and the trajectory is terminated at a range of 3590 yards. Sixty data points were taken at a 4-Hz rate.

g. Trajectory 7; medium-range, high speed, non-maneuvering

Trajectory 7 commences at a range of 5000 yards in a 20 degree dive with an initial velocity of -300 yds/sec. No maneuver is executed. The trajectory is terminated at a final range of 575 yards. Sixty data points were taken at a 4-Hz rate.

h. Trajectory 8; long-range, high speed, non-maneuvering

Trajectory 8 commences at a range of 8000 yards in a 20 degree dive with an initial velocity of -300 yds/sec. No maneuver is executed. The trajectory is terminated at a final range of 3575 yards. Sixty data points were taken at a 4-Hz rate.

i. Trajectory 9; long-range, low speed, non-maneuvering

Trajectory 9 commences at a range of 8000 yards in 20 degree dive with an initial velocity of -200 yds/sec. No maneuver is executed. The trajectory is terminated at the final sample range of 50 yards. One-hundred-sixty data points were taken at a 4-Hz rate.

j. Trajectory 10; long-range, low speed, moderate level maneuver

Trajectory 10 commences at a range of 8000 yards in a 20 degree dive with an initial velocity of -200 yds/sec. At 4000 yards the target executes a 2.0 g $\sin^2$ planar maneuver lasting 13.5 sec. The maneuver is completed at a range of 1200 yards and the trajectory is terminated at

the final sample range of 50 yards. One-hundred-sixty data points were taken at a 4-Hz rate.

k. Trajectory 11; long-range, low speed, moderate level maneuver

Trajectory 11 commences at a range of 8000 yards in a 20 degree dive with an initial velocity of -200 yds/sec. At 2700 yards the target executes a 2.0 g $\sin^2$ planar maneuver lasting 13.5 sec. The maneuver is completed at the final sample range of 50 yards. One-hundred-sixty data points were taken at a 4-Hz rate.

l. Trajectory 12; long-range, low speed, moderate level maneuver

Trajectory 12 commences at a range of 8000 yards in a 20 degree dive with an initial velocity of -200 yds/sec. At 3333 yards the target executes a 2.0 g $\sin^2$ planar maneuver lasting 13.5 sec. The maneuver is completed at a range of 500 yards and the trajectory is terminated at the final sample range of 50 yards. One-hundred-sixty data points were taken at a 4-Hz rate.

m. Trajectory 13; long-range, low speed, moderate level maneuver

Trajectory 13 commences at a range of 8000 yards in a 20 degree dive with an initial velocity of -200 yds/sec. At 2700 yards the target executes a 2.5 g $\sin^2$ planar maneuver lasting 10 sec. The maneuver is completed at the final sample range of 50 yards. One-hundred-sixty data points were taken at a 4-Hz rate.

n. Trajectory 14; long-range, low speed, moderate level maneuver

Trajectory 14 commences at a range of 8000 yards in a 20 degree dive with an initial velocity of -200 yds/sec. At 4000 yards the target executes a 2.5 g $\sin^2$ planar maneuver lasting 10 sec. The maneuver is completed at a range of 600 yards and the trajectory is terminated at the final sample range of 50 yards. One-hundred-sixty data points were taken at a 4-Hz rate.

B.2 TRAJECTORY GENERATING PROGRAM

The trajectories of Section B.1 are the products of a tracking generating program written by Parr. A complete description of its use, development of the planar $\sin^2$ maneuver, program listing and several examples may be found in Parr's work [Ref. 4].

APPENDIX C

THE N-STEP PREDICTION PROBLEM

The ultimate test of any fire control system is its ability to destroy an engaged target. To this end, the determining factor is the accuracy with which it predicts the future position of the target, since launcher laying orders are derived from knowledge of the target's present and future states. This of course, neglects the response characteristics of the launcher and the weapon (projectile) ballistics, both of which are assumed here to have negligible effects on the end result.

The interval over which the predictions are made is derived from a straight line approximation of the plot of projectile Time of Flight (TOF) vs. the target's present slant range, for a 5"/54 CAL. projectile. For the interval in question, $\{\text{TOF}: \text{TOF} \varepsilon (0.0, 10.0)\}$ the straight line approximation takes on the value

$$\text{TOF} = 1.43 \times 10^{-3} \text{ (sec/yd)} \times \text{(slant range)} \qquad \text{(C.1)}$$

The TOF interval, (0.0,10.0) was based on the assumption that the maximum effective range of the 5"/54 CAL gun against an air target is 7000 yds, corresponding to a TOF of approximately 10.0 seconds.

In the absence of any deterministic inputs to the system, the target's future position predicted over N samples is given by

$$\hat{\underset{\sim}{x}}(K+N/K) = \underset{\sim}{\phi}^{N}\hat{\underset{\sim}{x}}(K/K) \qquad (C.2)$$

Raising the state transition matrix $\underset{\sim}{\phi}(K,K+1)$ to a decimal power presents some computational difficulties, so N is formed by computing the TOF from $\hat{x}_1(K/K)$ (the current estimate of the target's slant range) then truncating this value to give the integer N. For large N the difficulty of raising a (9 x 9) matrix to the $N^{\underline{th}}$ power still exists. Note, however, that the assumption of linear plant dynamics decouples the filter in each of three coordinate directions. Partitioning and decoupling we have for the system,

$$\underset{\sim}{x}(K+1) = \begin{bmatrix} \underset{\sim}{\phi}_{11} & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{0}_{22} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{0}_{33} \end{bmatrix} \begin{bmatrix} \underset{\sim}{x}_1(K) \\ \underset{\sim}{x}_2(K) \\ \underset{\sim}{x}_3(K) \end{bmatrix} + \begin{bmatrix} \underset{\sim}{\Gamma}_1 & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{\Gamma}_2 & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{\Gamma}_3 \end{bmatrix} \begin{bmatrix} \underset{\sim}{w}_1(K) \\ \underset{\sim}{w}_w(K) \\ \underset{\sim}{w}_3(K) \end{bmatrix} \qquad (C.3)$$

where

$$\begin{aligned} \underset{\sim}{x}_1(K) &= [r,\dot{r},\ddot{r}]^T & \underset{\sim}{w}_1(K) &= [w_1(K)] \\ \underset{\sim}{x}_2(K) &= [\phi,\dot{\phi},\ddot{\phi}]^T & \underset{\sim}{w}_2(K) &= [w_2(K)] \qquad (C.4) \\ \underset{\sim}{x}_3(K) &= [\theta,\dot{\theta},\ddot{\theta}]^T & \underset{\sim}{w}_3(K) &= [w_3(K)] \end{aligned}$$

Further, note that for each of the models discussed in this thesis, each of the $\underset{\sim}{\phi}_{ii}$ i = 1, 2, 3 are lower triangular matrices. Following the method given by Parr [Ref. 4], the elements of the $\underset{\sim}{\phi}_{ii}$ i = 1,2,3 are operated on individually and the resulting $\underset{\sim}{\phi}_{ii}^{N}$ i = 1,2,3 are adjoined to form the system state transition matrix $\underset{\sim}{\phi}^{N}(K+N,K)$. The intermediate result follows quickly and is shown on the following page.

$$\underset{\sim}{\phi}_{ii}^{N} = \begin{bmatrix} \phi_{ii}(1,1) & (\phi_{ii}(1,2))N & \left\{\phi_{ii}(1,3) \sum_{M=1} \phi_{ii}(3,3)^{M-1} \right. \\ & & \left. + \ \phi_{ii}(1,)\phi_{ii}(2,3) \sum_{M=1}^{N-1} (N-m)\phi_{ii}(3,3)^{M-1} \right\} \\ 0 & \phi_{ii}(2,2) & \phi_{ii}(2,3) \sum_{M=1}^{N} \phi_{ii}(3,3)^{M-1} \\ 0 & 0 & \phi_{ii}(3,3)^{N} \end{bmatrix} \qquad (C.5)$$

Finally, adjoining the $\underset{\sim}{\phi}_{ii}^{N}$'s gives

$$\underset{\sim}{\phi}^{N}(K+N,K) = \begin{bmatrix} \underset{\sim}{\phi}_{11}^{N} & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{\phi}_{22}^{N} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{\phi}_{33}^{N} \end{bmatrix} \qquad (C.6)$$

The result shown in Equation (C.5) is easily programmed and involves simple manipulations without the need for extensive matrix multiplication. Note that the result shown in Equation (C.5) is equivalent to (for the single dimension)

$$r(K+N) = r(K) + \dot{r}(K)[(N-K)T] + \frac{1}{2}\ddot{r}(K)[(N-K)T]^2 \qquad (C.7)$$

For the case of the constant-velocity model, $\phi_{ii}(1,3)$, $\phi_{ii}(2,3)$, and $\phi_{ii}(3,j)$ $j = 1,2,3$ are set to zero. The prediction is then easily found by using Equation (C.2).

The magnitude of the N-step prediction error is defined to be the magnitude of a position vector, say $\overrightarrow{ERRNET}$, drawn between the estimate at time $t = (K+N)T$, $\hat{\underset{\sim}{x}}(K+N/K)$, and the reference trajectory, $\underset{\sim}{x}(K+N)$

$$||\overrightarrow{ERRNET}|| = ||\underset{\sim}{x}(K+N) - \hat{\underset{\sim}{x}}(K+N/K)|| \qquad (C.8)$$

Note that no reference is made to the direction of the error vector since the previously defined kill region is a sphere of constant radius centered at the reference position $\underset{\sim}{x}(K+N)$.

The primary interest lies in the following three regions

1) $||\overrightarrow{\mathrm{ERRNET}}|| \leq 8.0$ yds hit

2) $8.0 < ||\overrightarrow{\mathrm{ERRNET}}|| \leq 16.0$ yds. scare

3) $||\overrightarrow{\mathrm{ERRNET}}|| > 16.0$ yds miss

To avoid the necessity for coordinate conversion, the error vector is determined by a simple rotation of the polar plane followed by applications of the law of cosines. Consider the situation depicted in Figure 4, where points a and b represent $\hat{\underset{\sim}{x}}(K+N)$ and $\underset{\sim}{x}(K+N/K)$ (position only). Then from Equation (C.8)

$$||\overrightarrow{\mathrm{ERRNET}}|| = ||\overrightarrow{ab}|| = ||\overrightarrow{ba}|| \tag{C.9}$$

If the polar plane in Figure 4 is rotated through some angle β about an axis constructed by drawing a line through the origin connecting the points (090,270) (keeping r_1 and r_2 fixed) Figure 5 is obtained. The angle of rotation is defined as

$$\beta = \min(\theta_1, \theta_2) \tag{C.10}$$

Point now lies on the rotated polar plane, the position of point b remains fixed, and point c represents the fixed range projection of point b onto the polar plane. Some rather obvious geometric relationships follow immediately;

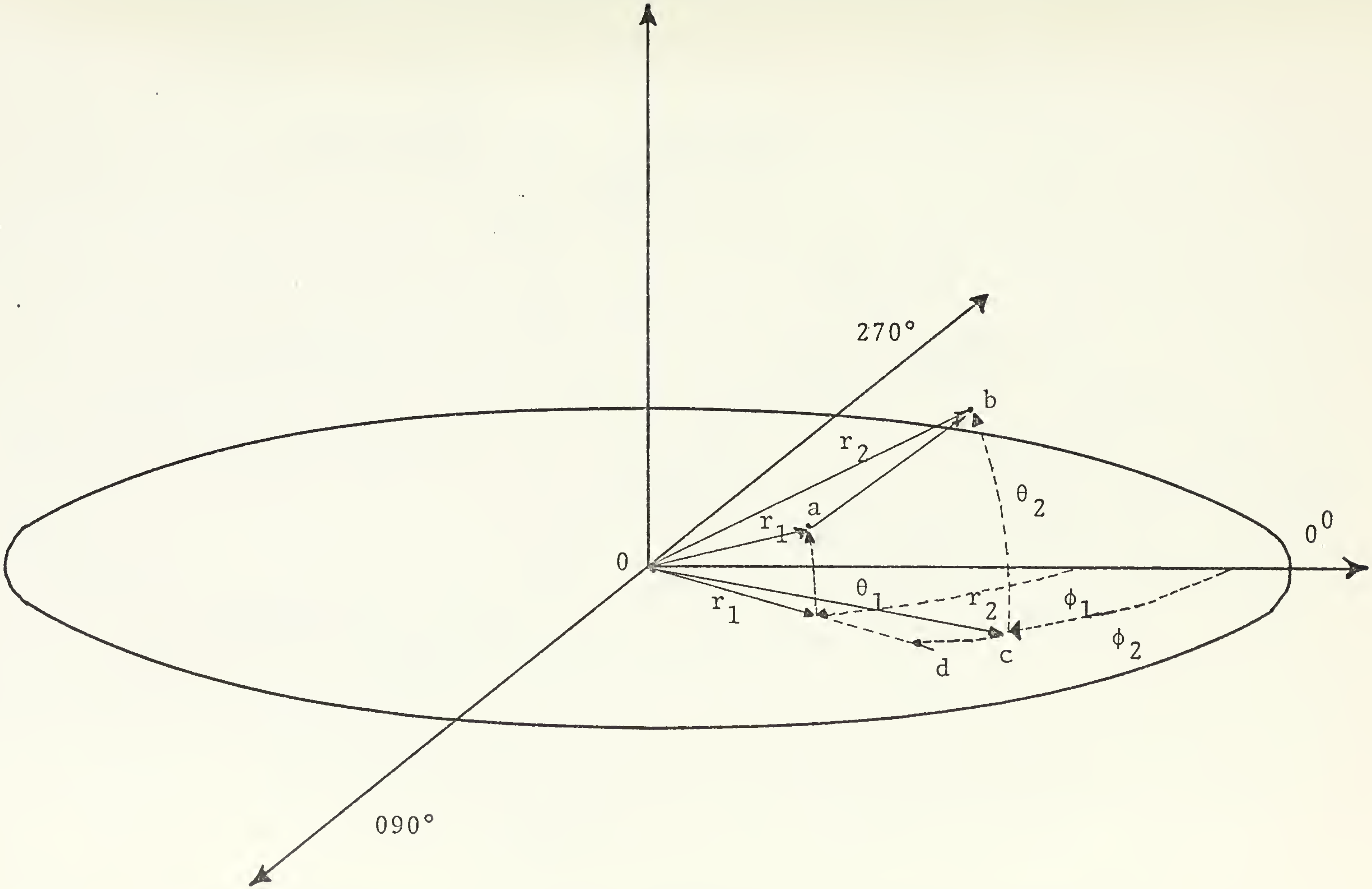


Fig. 4. GEOMETRY OF THE PREDICTED POSITION ERROR PROBLEM IN A SPHERICAL COORDINATE SYSTEM

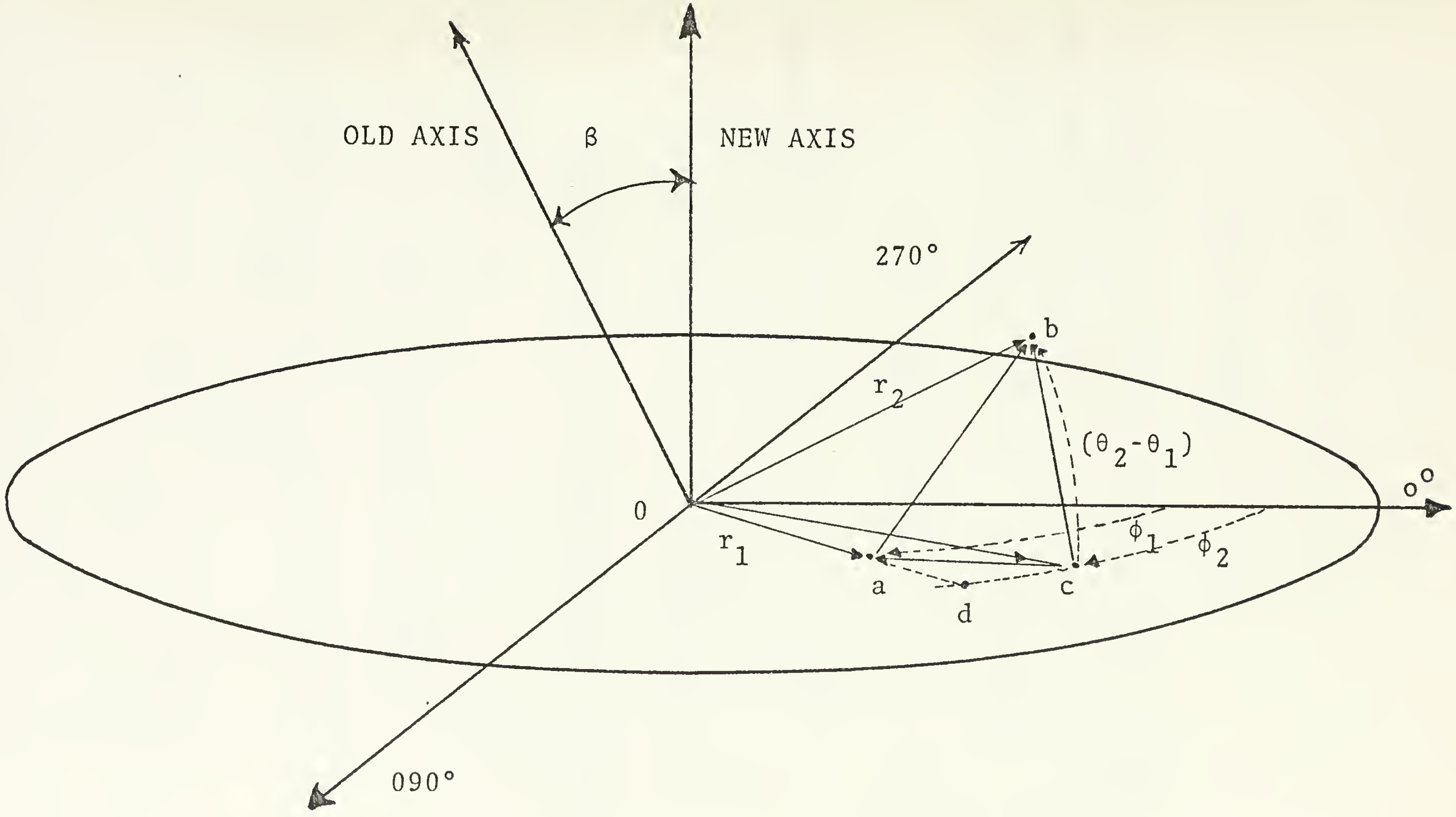


Fig. 5. THE POLAR PLANE AFTER ROTATION

$$\Delta\theta = \underline{/boc} = |\theta_1 - \theta_2|$$

$$\Delta R = |\overrightarrow{ad}| = |r_1 - r_2| \tag{C.11}$$

$$|\overrightarrow{od}| = \max(r_1, r_2)$$

The problem then remains of finding |ab|. Figure 6 shows the projection onto the rotated polar plane. Note that triangle cod is an isosceles triangle, then

$$\underline{/cdo} = \frac{180 - \Delta\phi}{2} \tag{C.12}$$

Applying the law of cosines $|\overrightarrow{ac}|$ is given by

$$|\overrightarrow{ac}| = \{(|\overrightarrow{ad}|)^2 + [(2.0)(\sin\frac{\Delta\phi}{2.0})(|\overrightarrow{od}|)]^2 - (2.0)(|\overrightarrow{ad}|)(2.0)(\sin\frac{\Delta\phi}{2.0})(|\overrightarrow{od}|)\cos\underline{/cdo}\}^{1/2} \tag{C.13}$$

$|\overrightarrow{bc}|$ is given by

$$|\overrightarrow{bc}| = (2.0)(|\overrightarrow{od}|)(\sin\frac{\Delta\theta}{2.0}) \tag{C.14}$$

Again applying the law of cosines and noting that $\underline{/bcd} = \underline{/bco}$, $|\overrightarrow{ab}|$ is given by

$$|\overrightarrow{ab}| = \{|\overrightarrow{bc}|^2 + |\overrightarrow{ac}|^2 - (2.0)(|\overrightarrow{bc}|)(|\overrightarrow{ac}|)\cos(\underline{/bcd})\}^{1/2} \tag{C.15}$$

Thus, the magnitude of the N-step prediction error has been determined without the need for any coordinate conversion. The validity and accuracy of this means of determining the

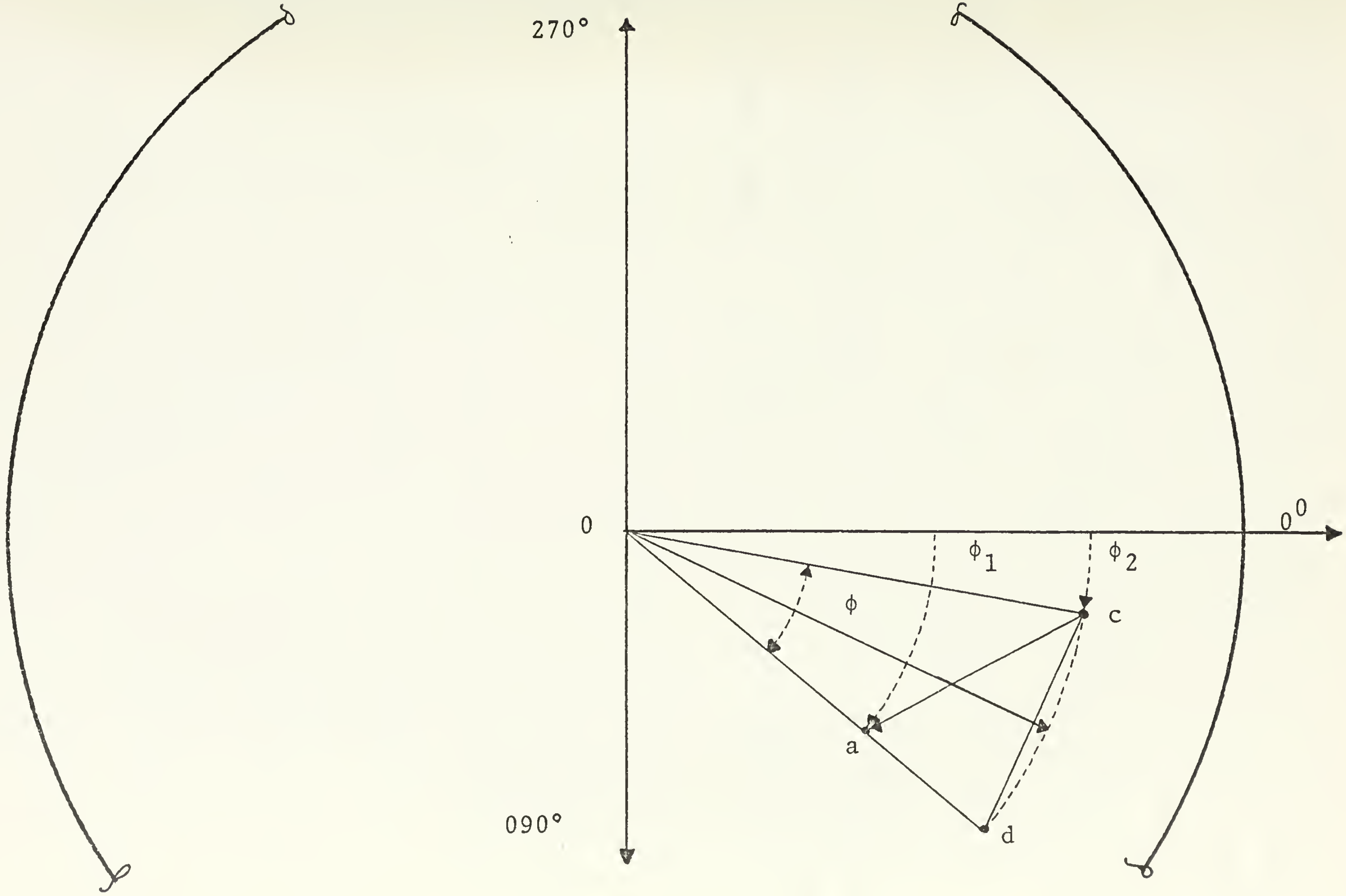


Fig. 6. FIXED RANGE PROJECTION ONTO THE ROTATED POLAR PLANE

error magnitude was verified by conversion of $\underset{\sim}{x}(K+N)$ and and $\hat{\underset{\sim}{x}}(K+N/K)$ to the Cartesian Coordinate system and determining the magnitude of the error vector by standard means. In every case investigated, the results given by the two different methods were in complete agreement. Again note that no attempt is made to determine the direction of the error vector $|\overrightarrow{ab}|$ since the regions of interest specify concentric spheres of constant radii centered at $\underset{\sim}{x}(K+N)$. The directionality of the error vector is eliminated in SUBROUTINE KILL through the use of the absolute values of the differences in bearing and elevation, and by using the maximum value of the actual and estimated ranges.

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